On the Feasibility of Ad-Hoc Localization Systems

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Abstract

Ad-hoc localization systems enable nodes in a sensor network to fix their positions in a global coordinate system using a relatively small number of *anchor nodes* that know their position through external means (*e.g.*, GPS). Because location information provides context to sensed data, such systems are a critical component of many sensor networks and have therefore received a fair amount of recent attention in the sensor networks literature. The efficacy of these systems is a function of the density of deployment and of anchor nodes, as well as the error in distance estimation (*ranging*) between nodes.

In this paper, we examine how these factors impact the performance of the system. This examination lays the groundwork for the main question we consider in this paper: Can the ability to estimate *bearing* to neighboring nodes greatly increase the performance of ad-hoc localization systems? We discuss the design of ad-hoc localization systems that use range together with either bearing or imprecise bearing (such as sectoring) information, and evaluate these systems using analysis and simulation.

1 Introduction

Sensor network localization has been an active area of research for the last few years. For sensor networks, and more generally for networks of embedded systems, the ability for nodes to determine their position through automatic means is recognized as an essential capability. The community has made great strides in ranging technologies, systems for infrastructured-based localization, and algorithms and techniques for ad-hoc localization (Section 2). This last class is the subject of this paper.

In an ad-hoc localization system, nodes determine their position in a common coordinate system using a number of *anchor nodes* that already know their location (through some external means, such as GPS [6]) in that coordinate system. These systems assume all nodes possess a *ranging* capability (the ability to estimate distances to other nodes). Using their range estimates, nodes use one of several *distributed position fixing* techniques to determine their positions in the coordinate system.

There are two characteristics that are highly desirable in a distributed ad-hoc localization system¹; in fact, we assume that these are design *requirements* for such systems. The fi rst requirement is that the performance of such a system be relatively insensitive to anchor placement, as long as the anchors are not placed in a degenerate confi guration. From a sensor network perspective, this is desirable since it may often be difficult to engineer anchor placements in the environments that these networks are deployed. Another way of saying this is that an ad-hoc localization system permits unplanned anchor placement. A second requirement is that relatively few anchors be necessary for obtaining good localization performance. This requirement is motivated by the fact that in some environments it may be difficult to obtain position estimates through external means (*e.g.*, because GPS signals can be significantly through foliage). We argue that ad-hoc localization systems should work well with *an order of magnitude* fewer anchors than nodes. This rule-of-thumb is motivated by a systems argument; if one in two or three nodes are required to be anchors, it would significantly constrain the deployment of such systems.

We begin this paper (Section 3) by evaluating the performance of a range of ad-hoc localization techniques proposed in the literature. The performance of ad-hoc localization depends upon several factors: the accuracy of ranging,

¹Our focus in this paper is on distributed ad-hoc localization systems. Henceforth, when we use the term "ad-hoc localization system", or simply "localization system", we mean this class of systems.

the density of node placement, the relative density of anchors, as well as the particular position fixing schemes in use. Using both analysis and extensive simulations, we find that ad-hoc localization systems begin to perform acceptably only at node densities well beyond the density required for network connectivity. We find this to be rather pessimistic—being required to deploy more resources to get a component of a system working seems undesirable from an architectural standpoint. Moreover, we argue that this is a fundamental limitation of ad-hoc localization systems that use ranging devices only, rather than a shortcoming that can be remedied by designing better localization schemes.

We then consider whether adding the ability to estimate *bearing* to neighboring nodes can *qualitatively* improve the performance of ad-hoc localization schemes (Section 4). We show that there exists a highly accurate position fi xing scheme that uses both range and bearing information in order to localize nodes, at node densities comparable to that required for network connectivity. This is obviously an idealization, since it is unclear if accurate bearing estimation devices can be built at the form factors and energy-levels that sensor network nodes require.

What is more feasible, perhaps, is the ability to *approximately* detect bearing. Guided by this observation, we examine whether devices that enable nodes to place neighbors within *sectors* can enable acceptable ad-hoc localization performance at node densities that are sufficient for connectivity (Section 5). We show that there exists a simple iterative scheme that can provide attractive ad-hoc localization performance using both range and sector estimates. We conclude the paper by arguing that, given the importance of node localization in a sensor network, the community should invest some effort in sector estimation devices that will enable practical ad-hoc localization systems.

2 Related Work

In recent years, there has been significant work in localization for sensor networks and networks of embedded devices. Localization has also been an active area of research within the robotics community for several years now. We briefly review the current state of knowledge in localization systems. Our intent is not to be exhaustive in our coverage of the literature; rather, we list representative pieces of work that help put this paper in context.

Ranging An important focus of the localization literature has been robust techniques for estimating distances between nodes (*ranging*). The networking community has focused on two classes of ranging techniques: RF-based ranging and acoustic ranging. For the purposes of this paper, the exact choice of ranging technique is not important, but we include a discussion of these techniques because, as we shall see, they do have some bearing on our discussion of ad-hoc localization.

RF-based ranging, as exemplified by the SpotON [10] and Calamari [21] systems, is based on the premise that by measuring received signal strength a receiver can determine its distance to a transmitter. This presumes that RF propagation in an environment can be accurately characterized by a simple path loss model, whose parameters are known. Using this technique, nodes can estimate distances to all neighbors within radio range. Range errors upwards of 10% have been reported in the literature [21], usually after a fairly involved calibration step that estimates the path loss parameters and adjusts for variations in transceiver characteristics.

A second class of ranging schemes is based on measuring the time-of-flight of an acoustic or ultrasound signal [8, 19]. More precisely, these techniques measure the difference in arrival times of simultaneously transmitted radio and ultrasound signals, then estimate distances knowing the speed of sound. Some approaches in acoustic ranging use spread spectrum approaches for resilience to multipath, and employ techniques to correct for latencies induced by other system components [8]. Such techniques provide an order or magnitude better accuracy (1-2% error) over distances of 3-6 meters (*i.e.*, signifi cantly lower than the nominal radio range of sensor platforms such as the Mica motes).

Finally, we note that the robotics community has, for some time now, used highly accurate laser range-finders (which exhibit less than 0.1% ranging error). It is unclear, however, whether such devices can be manufactured in the form factors and energy-consumption levels required of sensor nodes.

Position Fixing A large body of work has examined algorithms for ad-hoc localization schemes. We taxonomize this literature later (Section 3.1), and only briefly introduce the related work here. Perhaps the earliest pieces of work in the area of sensor network localization can be attributed to Niculescu *et al.* [12] and Savvides *et al.* [20, 19]. In the former, nodes fi rst estimate their distances to anchors using one of several techniques (DV-hop, DV-distance, and a Euclidean scheme), then fi x their own position using these distances. The latter proposes an elegant *N*-hop multilateration scheme which we discuss later. That work also discusses a Kalman fi Itering based position refi nement phase to

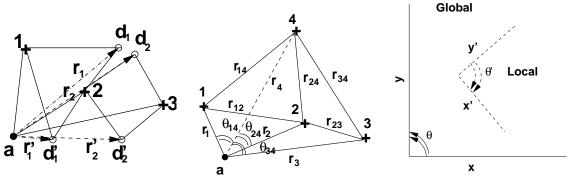


Figure 1: Voting scheme

Figure 2: Lateration

Figure 3: Global and local coordinate systems

improve position estimates. In more recent work, Savvides *et al.* discuss the error characteristics and the dependence on network size and anchor density of their schemes [18]. Savarese *et al.* [17] propose a three-phase scheme, where they obtain crude position estimates and use an iterative multi-lateration based scheme to refi ne the estimates. Each node uses the locations of its neighbors to multi-laterate and re-estimate its position. Finally, Langendoen *et al.* [11] discuss a fairly detailed comparison of all the above schemes in the face of ranging errors, different node densities and anchor fractions. Our work draws heavily upon theirs.

Somewhat tangentially related to our work is other literature on localization. Examples of such work include: position inference using specialized beacons [3], optimization methods for centrally estimating locations [5], in-building localization systems that enable position for context-aware applications [14, 9, 2], systems for estimating orientation of handheld devices [15], and systems for placing nodes within a relative coordinate system [4]. We do not survey this literature in detail.

Finally, predating the work in sensor network localization, there exists significant work in robotic node localization. This has discovered many tools that are applicable to ad-hoc localization systems: Kalman filter approaches to cooperative localization [16], refinement of probabilistic estimates of robot positions [7], and maximum-likelihood estimation of relative robot positions [1]. While not directly relevant to our work, they have influenced the design of some of our localization schemes.

3 Ad-Hoc Localization Using Ranging

As we have seen in Section 2, most of the *distributed* ad-hoc localization schemes studied till date leverage the ability of nodes to estimate distances to other nodes using ranging. Our goal in this section is to fairly extensively evaluate, through analysis and simulation, a representative sampling of such ad-hoc localization schemes, with the intent of understanding their error characteristics as a function of node and anchor density. Based on the simulations we gain some insight into the fundamental limitations (qualitatively and quantitatively) of range-only methods. In the rest of this section, for conciseness and to distinguish this class of localization approaches from those we consider later in the paper, we use the term *r*-only localization to describe these methods.

In some recent work, Langendoen *et al.* [11] have performed similar analyses. However, our evaluations (presented in this section) differ from theirs in several respects. In our simulations, we use larger topologies, examine a wider range of density and anchor ratios, and use a slightly more sophisticated ranging error model. We also introduce some improved localization schemes of our own into the mix of schemes, and examine more metrics to explore the effi cacy of ad-hoc localization.

3.1 A Taxonomy of Ad-Hoc Localization Schemes

Before describing the schemes we evaluate, we taxonomize *r*-only localization methods. Following Langendoen *et al.* [11], such localization schemes can be divided into three distinct sub-problems or stages: estimating distance to anchors; getting an initial position estimate; and iteratively refi ning the position estimate. When only ranging techniques are available, a sensor needs to estimate its distance to at least three anchors in order to localize itself into a

global coordinate system common to those anchors. All *r*-only localization schemes begin with each sensor trying to estimate its distance (or some approximation thereof) to anchors — the first sub-problem. Having obtained distances to three anchors, obtaining an initial position estimate reduces to a simple geometric problem commonly referred to as *lateration*. With these estimates, some *r*-only localization approaches incorporate a refi nement scheme that improves position estimates in the face of ranging error.

In the following subsections, we describe each of these three stages, and discuss the r-only localization schemes we evaluate.

Estimation of distance to an anchor

Broadly speaking, there are two classes of approaches to estimating distance to anchors: *topological* and *geometric*. Topological techniques are content to roughly estimate distance to anchors using information obtained by message flooding. Geometric techniques, on the other hand, more carefully compute distances to anchors and generally exhibit higher accuracy.

One example of a topological approach is what Niculescu *et al.* [12] the *DV-dist* approach. In this approach, each anchor initiates a (possibly constrained) flood. As the flooding message propagates, it accumulates the range measurement corresponding to each hop. Each node measures its distance to an anchor as the minimum accumulated distance. This approach consistently over-estimates the distance to anchors and is subject to accumulation of error over multiple hops. A slight variant of this approach estimates distance using hop-counts to anchors, a technique called DV-hop [12]. In Section 3.3, we evaluate an ad-hoc localization scheme using the DV-dist approach.

Geometric approaches are more sophisticated, and can be classified into: those that use relative bearing information when geometrically computing distance to anchors, and those that do not.

An example of the latter is the Euclidean approach proposed by Niculescu $et\ al.$ [12]. The scheme is illustrated in Figure 1. Essentially, using its neighbors' computed distances to an anchor, a node determines various possibilities for its own distance to the same anchor, and picks the most likely using a 'voting' scheme. For example, Node d has three nodes (1, 2, and 3) which have their estimates of distance (and positions) from anchor node a. Each pair of nodes gives raise to two possible values of ranges. For example, the pair (1,2) gives raise to the two possible ranges r_1 (for d_1) and r_1' (for d_1'), while the pair (2,3) gives rise to r_2 (for d_2) and r_2' (for d_2'). Under error free conditions, d_1 and d_2 would be identical, while d_2 and d_2' would not and so r_1 would be the true range. With ranging subject to errors, the true solutions may not overlap, but lie 'close' to each other as illustrated by d_1 and d_1' in Figure 1. In the voting based scheme, each solution votes for solutions in its vicinity and the solution with the maximum votes is chosen as the true solution. We include this scheme in our evaluations as well. In our implementation of this voting scheme, a solution from one pair of nodes votes to the closer (in terms of Euclidean distance) of the two solutions from another pair with a vote whose strength is equal to the negative of the difference between them.

Finally, the last (and most sophisticated) distance estimation technique is a geometric technique that implicitly computes relative bearing to neighbors (in addition to distance from an anchor) [19]. This technique is based on *relative localization*, as illustrated in Figure 2. In this fi gure, nodes 1, 2 and 3, know their ranges to the anchor node 0 and are within ranging distance of each other (r_{12} , r_{23} , r_{13} can be measured). Node 4 has the three nodes 1, 2 and 3 in its ranging radius (can measure r_{14} , r_{24} , r_{34}) and hence can calculate its distance to the anchor a. With that information, node 4 can also determine the relative angles θ_{34} , θ_{24} , θ_{14} . Thus, nodes 1, 2, 3 and 4 can estimate their positions in a local coordinate system. A set of relatively localized nodes are localized within a local coordinate system, with its origin as the anchor. In general, this coordinate system may be a translated, rotated and mirrored (relative angles being measured in the opposite sense) version of the global frame of reference as in Figure 3. The only absolute information (invariant in the global reference) obtained from relative localization with respect to an anchor are the ranges of a node from the anchor. Knowing how three or more of its neighbors place themselves in a local coordinate system with respect to an anchor, a node can place itself in that same relative coordinate system using *multi-lateration*. In Section 3.3, we evaluate an ad-hoc localization scheme that uses relative localization.

Initial position estimate and refinement

Once a node has estimated its distance to three anchors, multi-lateration seems to be the most commonly used approach for obtaining the node's initial position estimate. The performance of multi-lateration depends on the distances and relative locations of the anchors and nodes. It works well if the anchors are far apart and enclose the nodes within their convex hull [18].

Finally, three different refinement schemes have been proposed in the literature. The first is iterative multilateration, where nodes continuously estimate their positions using the estimated locations and ranges of their neighbors, and hoping to improve their locations [20]. The second is a Kalman filter based approach, which estimates the Jacobian of the multi-lateration equations in an attempt to obtain a Gaussian representation for the error in the solution [19]. The third approach is a heuristic proposed in [17] where bounding boxes for the solutions are iteratively shrunk.

In the following section, we describe a distributed refi nement scheme which explicitly attempts to find locations for the nodes which 'best' fits (in a least-mean-squares sense) the set of all measurements made in the sensor network. This scheme has independent interest, but also lays the groundwork for some localization schemes we introduce later in the paper.

Iterative Least-Mean-Squared Refinement

Our proposed refi nement scheme attempts to find locations for the sensors, which best explains (in terms of weighted mean squared error), the set of range measurements taken over the entire network. A range measurement r_{ij} taken at a node n_i to measure range to another node n_j , gives an equation,

$$\sqrt{(x_i - x_j)^T (x_i - x_j)} - r_{ij} = 0.$$
 (1)

Here, $x_i, x_j \in \Re^2$ are the *locations* of nodes n_i and n_j . For every measurement taken in the network we would then have one such equation. Since the number of measurements (hence the number of equations) can far exceed the number of unknown locations, we have an over-determined system. A common way to solve such an over-determined system is to find locations which 'best' fit the set of equations. Since each measurement could potentially have different error characteristics, some measurements could be deemed more reliable than others. In such a scenario, preferentially weighting the reliable equations in the equation set could give improved solutions. For this we adopt the most commonly used method for finding a best fit for locations—we find locations that *minimize the weighted mean square error taken over all the equations*.

We label such a scheme LMSR. LMSR basically weights measurements with their observed σ_{ij} , since these are a direct measure of the certainty of the measurements. As we describe below, LMSR is amenable to a distributed implementation. The theory behind LMSR assumes that ranging errors are Gaussian with known standard deviations σ_{ij} corresponding to a range measurement r_{ij} . This assumption is not critical to the scheme but we do find from measurements (see below) that the Gaussian assumption holds for range errors. Furthermore, in an implemented system, we would expect to use the nominal standard deviation obtained from a data sheet, standard deviations obtained by calibrating the devices prior to measurement, or the standard deviations of multiple range measurements.

We now describe the theory behind LMSR. Let G = (N, E) be a graph representing the connectivity of the sensor network, N be the set of all nodes, and $E = e_{ij}$ the set of all edges. An edge exists between two nodes which are within ranging neighborhood of each other. Further, let A_i represent the set of nodes that node n_i can range to. Also let $L \subset N$, be the set of all anchors, hence, the x_i s for $n_i \in L$ are known, and the remaining x_i s are unknown.

Formally, LMSR attempts to minimize the objective function,

$$J = \sum_{e_{ij} \in E} \sigma_{ij}^2 \left(\sqrt{(x_i - x_j)^T (x_i - x_j)} - r_{ij} \right)^2.$$
 (2)

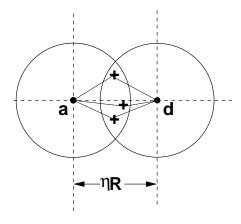
The conditions for local minima can be written as, the set of |N| - |L| simultaneous equations,

$$\sum_{n_j \in A_i} \left(\sigma_{ij}^2 + \sigma_{ji}^2 - \frac{\sigma_{ij}^2 r_{ij} + \sigma_{ji}^2 r_{ji}}{\sqrt{(x_i - x_j)^T (x_i - x_j)}} \right) (x_i - x_j) = 0, \ \forall n_i \in N - L, n_j \in N.$$
 (3)

The above equations can be re-written as,

$$x_{i} = \frac{\sum_{n_{j} \in A_{i}} \left(\sigma_{ij}^{2} + \sigma_{ji}^{2}\right) x_{j} + \frac{\sigma_{ij}^{2} r_{ij} + \sigma_{ji}^{2} r_{ji}}{\sqrt{\left(x_{i} - x_{j}\right)^{T} \left(x_{i} - x_{j}\right)}}}{\sum_{n_{j} \in A_{i}} \sigma_{ij}^{2} + \sigma_{ji}^{2}}, \ \forall n_{i} \in N - L, n_{j} \in N.$$

$$(4)$$



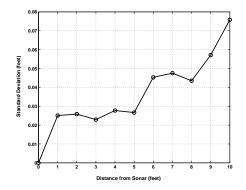


Figure 4: 3 Connectedness

Figure 5: Sonar ranging error std as a fuction of range

Note that Equation 4 depends *only* on the locations estimates of nodes within x_i 's ranging neighborhood and the ranges they measure. This allows for a distributed implementation of LMSR, where nodes continuously exchange their current location estimates, then use Equation 4 to update their estimates until they converge. LMSR will converge to the nearest local minima in the objective function, hence an initial guess 'sufficiently' close to the global minimum will force the nodes to a least-mean-squared fit of the measurements. This initial guess is, of course, obtained from the second stage of *r*-only localization.

3.2 A Simple Analytical Comparison of the Schemes

Having described the various r-only localization schemes, we now attempt to analyze the schemes and try to predict their performance. We then solidify these discussions with a quantitative evaluation using simulation of some of these schemes in Section 3.3.

Estimating distances from anchors: The geometric schemes use ranges and locations of three or more nodes to relatively locate (or determine range to an anchor) a new node. This imposes a requirement that a node have at least three nodes within its ranging neighborhood and further that these nodes be connected among themselves (Figure 4). We call this property 3-connectedness to an anchor. Let the set of all nodes 3-connected to an anchor $a \in N$ be denoted by $N_a^3 \subset N$. Formally, $n_i \in N_a^3$ if and only if there exists $n_j, n_k, n_l \in N_a^3 \cap A_i$ such that the sub-graph comprising nodes n_j, n_k, n_l is connected.

Intuitively, 3-connectedness is a strong requirement of sensor deployment topology, far stronger than mere connectedness of the overall topology. As we now argue (using a simple geometric model), 3-connectedness *may leave many nodes unable to find an initial position fix even at moderate node densities*. Figure 4 shows a typical scenario, where, in order to obtain a range estimate from *a*, node *d* needs 3 nodes in the region of intersection of the two circles (representing measurement neighborhoods of *a* and *d*) shown. Let *R* be the radius of the ranging neighborhood.

If the node d is at a distance ηR from the anchor a, the area of overlap is:

$$O(\eta R) = R^2 \left(\cos^{-1} \left(\frac{\eta}{2} \right) - \frac{\eta}{2} \sqrt{\left(1 - \frac{\eta^2}{4} \right)} \right). \tag{5}$$

An average of 3 nodes in this area implies an average node density of

$$p_e(\eta) = \frac{3\pi}{\pi \cos^{-1}\left(\frac{\eta}{2}\right) - \frac{\eta}{2}\sqrt{\left(1 - \frac{\eta^2}{4}\right)}}\tag{6}$$

in the ranging neighborhood. Nodes beyond one hop from the anchor have $\eta > 1$ and $\rho_e(1) = 7.7$. This implies that any geometric scheme will require a deployment density of least 8 nodes within the ranging neighborhood for them to start working. Thus, it is expected that at densities below 8 almost none (or a very small fraction) of the nodes will be

localized. Notice that our argument implicitly assumes a reasonably low fraction of anchors, as is desirable for ad-hoc localization systems. For example, if the three nodes in Figure 4 already had obtained their position through external means (rather than inferring it from anchor *a*), our argument would not hold.

At a density of p, the expected distance μ between two nodes is:

$$\mu(p) = \sqrt{\frac{\pi}{p}}R. \tag{7}$$

This means that the expected distance of the closest node to a beyond the ranging neighborhood of a is:

$$\chi(p) = \left(\lfloor \frac{R}{\mu(p)} \rfloor + 1\right) \mu(p). \tag{8}$$

We wish to determine, the critical node density (p_c) , which roughly guarantees that almost all nodes in the network will be 3-connected. This will allow localization by geometric schemes at low anchor ratios. Suppose the node density is such that the closest node beyond the ranging neighborhood is 3-connected to the anchor. Then, this node can determine its relative position (or distance) and act as an anchor for the closest node beyond its ranging neighborhood and so on. Such a density will then will allow localization of almost all nodes at very low anchor ratios. This condition is given by the equation,

$$p_e\left(\frac{\chi(p_c)}{R}\right) = p_c. \tag{9}$$

Solving equation 9 gives $\rho_c \approx 14.9845$. Hence, 3-connectedness becomes highly likely for almost all nodes at densities around 14 to 15 and one would expect almost all the nodes to be localized (with very low error) at these densities and beyond. Furthermore, at low densities, those nodes that do obtain an initial position estimate get quite accurate estimates using geometric schemes. Our simulations in Section 3.3 bear this out.

By contrast, the topological schemes do not require 3-connectedness and the existence of a single network path to the anchor is sufficient to obtain distances to anchors. Using a similar analysis, the critical density can be calculated as,

$$\frac{1}{3}p_e\left(\frac{\chi(p_c)}{R}\right) = p_c,\tag{10}$$

since, the overlap region now requires only one node rather than three. The solution to equation 10 is $\rho_c \approx 5.9956$. Hence, it follows that connectedness will be highly likely at densities of 5-6 nodes per ranging neighborhood and beyond. One expects almost all nodes to be localized at and beyond these densities at low anchor fractions. Note that the above arguments do not depend on actual values of radio range or total number of nodes in the network, but only depend on the average number of nodes in the ranging neighborhood.

While the density requirements for topological approaches seem to be more relaxed, this comes at the cost of increased localization error. The ranges calculated beyond one hop are consistent over-estimates, sometimes by a very large margin. We quantify this tradeoff in Section 3.3.

Effect of refinement on the schemes: The geometric approaches, provide fairly accurate estimates for the range from anchors. This, coupled with such range estimates from several anchors provides a fairly accurate initial guess to work with. The unlocalized nodes do not provide an initial guess to work with and hence cannot be refined. As pointed out by Langendoen *et al.* [11], refinement does not improve performance significantly for geometric approaches.

On the other hand, the topological schemes for estimating distances to anchors leave enough room for improvement by refi nement. For example, in the *DV-dist* approach at least, the estimates of ranges to anchors is incorrect often resulting in highly inaccurate initial guesses.

3.3 Evaluating Ad-Hoc Localization Systems

In the previous sections, we taxonomized the various r-only ad-hoc localization schemes known in the literature, and examined analytically their performance as a function of node density. In this section, we use simulation to discuss the performance of r-only localization schemes.

The Schemes According to our taxonomy (Section 3.1), r-only localization can be classified into three phases. For the second of these phases, obtaining an initial position fix, almost all published schemes use multi-lateration. So do we

For the last of these phases, refi nement, we use the LMSR scheme described in Section 3.1. This choice is somewhat different from that published in the literature, but we are fairly confi dent that LMSR is representative of the benefits that refi nement can provide. For geometric schemes, refi nement provides very little benefit anyway. We found that, for topological schemes (*DV-dist*), the performance of our refi nement scheme 'closely' matched the results reported by [11]. For these reasons, we do not believe that evaluating other refi nements schemes will change the conclusions we make below.

The first, and most crucial, phase in range-only ad-hoc localization is estimating a node's distance to anchors. This phase presents the most choices. In our evaluations, we choose three schemes, representative of three different classes of our taxonomy: the *Euclidean* [12] geometric scheme that does not estimate relative bearing to anchors; the *relative localization* technique that implicitly computes relative bearing to anchors; and the *DV-dist* topological scheme. We believe that this set adequately represents the state of the art in range-only ad-hoc localization mechanisms.

Simulation Methodology All our simulations were carried out in a home-grown simulator. We carefully implemented many of the schemes described in the literature, and our results are in agreement with published results. Since our approach used an independent implementation, it represents a validation of most of the prior work in the field.

All our simulations use a field of 1000 randomly deployed nodes (node locations being drawn from a uniform distribution over the region). We chose the ranging distance to be 10m, and we also assume that this is the communication radius.

The density of the nodes (the number of nodes in a ranging neighborhood) was controlled by increasing the area of deployment while keeping the number of nodes 1000. For example, an $252m \times 252m$ region results in a density of 5, while a $200m \times 200m$ region results in a density of 8 nodes. A fraction of nodes (the anchor fraction) were chosen to be anchor nodes.

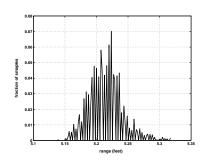
Most of these assumptions are consistent with the literature. One place where we deviate from the literature is in the use of a different model for ranging error. Most published work, at least in sensor network localization, has assumed that ranging error is independent of the ranging distance. The error model we use is a Gaussian with its standard deviation varying linearly with range, and given by $\sigma(r) = \alpha r$. Consider a uniform error model which uses a fixed standard deviation σ . In comparing this with our distance dependent error model where the standard deviation at the maximum distance was also σ , we see that our error model is more generous (or less pessimal) towards ad-hoc localization schemes; for smaller ranges, it assigns less ranging error. Our conclusions are not affected by this choice of error model; we have verified that using a uniform error model gives qualitatively the same result.

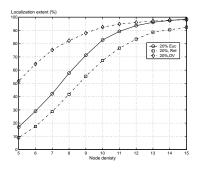
To validate our error model, we conducted measurements using a sonar ranging device on an off-the-shelf robot. Although the robot is significantly more capable that a sensor node, we anticipate that the error characteristics of the sonar will be similar to the ultra-sound ranging devices in consideration for sensor networks [8]. In fact, our error measurements in this device (about 1% over 3-4m) agrees qualitatively with ultrasound ranging error estimates measured elsewhere [8, 20]. Figure 5 shows the variation of standard deviation as a function of distance. Error variance increases with distance, and for the range we consider, a linear fit works reasonable well with an α of approximately 0.7.

Furthermore, the Gaussian assumption is, to a large extent, borne out by measurements. For example, we took measurements on a sonar ranging device at a 5 foot distance. The distribution of samples at this distance is shown in Figure 6 and is visually Gaussian with a slightly longer right tail. With increasing distance, this tail becomes slightly more pronounced.

Metrics Most of the literature focuses on either *RMS error* or *mean error* as measures of the *accuracy* of ad-hoc localization. Both measures define the error of localization as the distance between the computed location and the actual (ground truth) location. RMS error can be skewed by nodes with large localization error, and often gives pessimistic estimates of localization error. Mean error weighs all values of localization errors equally and hence provides a more realistic estimate of the expected value of error. Mean error is defined as,

$$e_{\mu} = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(x_i - \hat{x}_i)^T (x_i - \hat{x}_i)}.$$
 (11)





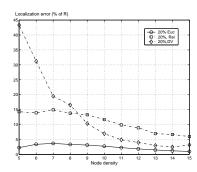


Figure 6: Sonar Ranging Measurement Distribution

Figure 7: Localization extent

Figure 8: Localization Error

where \hat{x}_i are the estimates of x_i found by the algorithm. In this paper, we use mean error as a measure of localization accuracy, both because it is less skewed than RMS error by large error values, but also because it is more easily understood as an error measure. We compute mean error only on nodes that were able to obtain a position estimate at all.

Also motivated by our discussion in Section 3.2, we introduce a new metric that measures the *extent* of localization. This is essentially a quantile metric and is defined as the fraction of non-anchor nodes that are localized to within 2 meters of their ground-truth position. In our simulations, 2m represents a fairly conservative estimate (about 20% of radio range). The extent metric serves to differentiate geometric schemes (which rely on 3-connectedness) and topological schemes (which do not). Much of the literature has ignored this metric, choosing to evaluate localization error only in regimes with high localization extent. We argue in Section 3.3 that this metric brings out a more complete picture of the dynamic performance range of ad-hoc localization schemes.

Simulation Results We have evaluated our three representative schemes using simulation for three different ranging errors (0.1%, 1% and 5%) and three different anchor fractions (5%, 10% and 20%). Given space constraints, we present results from one combination (20% anchors, 1% error) that represents the state of the art ranging error using ultrasound ranging, and a 20% anchor fraction that represents a fairly generous sprinkling of anchors. Other combinations of errors and anchor densities do not affect our conclusions of Section 3.4.

Figure 7 shows the localization extent for our three schemes. Qualitatively, there seem to be two classes of performance: the DV-dist approach (a topological scheme) has higher localization extent at lower node densities than the Euclidean and the relative localization schemes (the geometric schemes), but those two catch up at about a density of 12. Clearly, the 3-connectedness requirement greatly affects the performance of the second class; only 20 to 30% of the nodes are able to get a position fi x at all at low densities. In the DV-dist approach, even a low densities, more than 95% of the nodes are able to get a position fi x (as predicted by our analysis), but their estimates are very poor in quality (a greater than 20% error in position). Finally, notice that the simulations are consistent with our analysis: at densities of 14 or 15, the network becomes 3-connected.

The same two classes are visible when we examine the localization error (Figure 8). The geometric schemes exhibit low mean error (and also low RMS error – we have computed this metric but do not include it in our evaluations) across the entire range of densities because if a node gets a position estimate, it gets a good one. On the other hand, the topological schemes start with high error at low densities and achieve good performance only at densities of 9 or 10. There is some performance difference between the two geometric schemes (Euclidean outperforms relative localization), but we suspect that their performance could be made comparable by applying well-known optimizations. We did not do this since our intent was not to compare similar schemes, but understand *classes* of performance).

The qualitative conclusion we draw from this is that r-only localization schemes require 11-12 nodes within the ranging neighborhood in order to achieve 90% localization and 5% accuracy.

3.4 The Feasibility of Ad-Hoc Localization Systems

Our conclusion above represents a somewhat subjective choice (90% extent, 5% mean error). However, we believe these thresholds to be fairly generous from the perspective of a deployed system. Also our evaluations are fairly extensive and borne out by analyses. We argue that *this result is pessimistic*, because in order for r-only localization

schemes to work, almost *twice as many nodes are required as for mere connectivity* (it is well known that a radio network with uniformly distributed nodes is connected, with high likelihood, if it has an expected number of 5-6 neighbors per node). This is an architectural oddity, since it isn't clear one should *have* to deploy twice as many resources for only one component (albeit a very important one) of a system.

We should note that the literature has long hinted at this pessimism, preferring to evaluate their schemes at relatively high densities. However, we could not find an explicit statement of this pessimism, rather a general acceptance of the state of affairs. We do not mean to disparage existing pieces of work; many of the localization schemes we have referred to have very elegant and intricate algorithms, and we could not (despite trying) improve any of them in any significant way. Rather, we think our observation is *fundamental* of *r*-only localization and optimizations and improved refi nement schemes cannot change this. Euclidean schemes require 3-connectedness, which occurs only at high densities. Topological schemes on the other hand rely on the congruency between topological and physical distance for low error and this sets in only at high densities.

There are three counter-arguments to our position, which we now discuss.

Argument 1: Why not just deploy more anchors? This is a reasonable position, and when it is feasible to make 30% or more of the nodes as anchors, then with high likelihood r-only localization will begin to work well. We have seen this in our simulations. However, this goes against the basic rationale for ad-hoc localization systems (Section 1).

Argument 2: What's wrong with deploying more resources: we need them anyway for increased network lifetimes? On the surface, this is an attractive argument. From an architectural perspective, however, we believe that *requiring* deployments to be redundant so that ad-hoc localization schemes work is undesirable.

But, the question of higher density deployments is a bit more subtle than just that. Note that in our simulations (and this is consistent with current practice in the literature), when we say "density" we mean the number of nodes within ranging radius. In drawing our conclusions of the previous section, we *implicitly equated* ranging radius and communication radius. While this assumption is true of RF-based ranging, the ranging errors for this technique are quite high (10% even with signifi cant calibration [21]). The more accurate ultrasound ranging devices have a reported range of 3-6m [20, 8], while the current radios on the Mica motes already have a range of tens of meters and the new Chipcon radios are reported to have a greater range. This means that *r*-only schemes might, in theory, require impossibly high density of nodes within the communication radius. This is not a fundamental limitation, however, and there are many possible approaches to circumvent this: controlling the transmit power on the radios (which might be necessary if node deployments need to be fairly dense anyway for application reasons), using UWB localization (the current generation UWB localization methods are reputed to exhibit high error), or finding low-power ranging devices which can range at higher distances (this may be diffi cult: a fairly good off-the-shelf sonar, the Polaroid 6500, which can range up to 35 ft. requires a 2A current draw when taking measurements).

Argument 3: Random anchor placement makes no sense, so we should perhaps consider more constrained placements? While more constrained placements, like placement of anchors along the periphery of the network can improve localization error, we do not believe they can increase localization extent at low densities. For the geometric schemes, recall that the 3-connectedness argument does not require a certain number or placement of anchors (as long as the fraction of anchors is low); rather it is a more fundamental property of *node density*. Topological schemes rely on the congruency of topology and physical distance to get reasonable error, and again this cannot be improved by preferentially placing more anchors. We have also verified this using our simulations, and the graphs are omitted for brevity.

Having painted this rather pessimistic picture, we now ask: Is there an alternative solution?

4 Using Bearing Information for Higher Localization Extent

We have argued that *r*-only localization schemes exhibit fairly poor performance. In this section, we discuss whether ad-hoc localization schemes could exhibit improved localization performance (low mean error and high localization extent even at low node densities) if each node had the ability to estimate bearing to its neighboring nodes, in addition to range. To our knowledge, this question has not been considered in the literature before.

In this section, we first describe a distributed algorithm for range-and-bearing based ad-hoc localization—we use the term $r - \theta$ localization to describe this class of ad-hoc localization systems. We end this section with a discussion of the performance and practical feasibility of $r - \theta$ localization.

Iterative Least-Mean-Squares Refinement using Range and Bearing The key intuition for the efficacy of $r - \theta$ localization can be seen from a simple geometric argument. Suppose that a node n_i has computed its position x_i (note that, per our notation in Section 3.1 x_i is a *vector* that represents the node's position). Now, if n_i is able to measure its range (r_{ij}) and its bearing (θ_{ij}) to another nearby node n_i , then the coordinates of n_i would given by

$$x_{j} = x_{i} + \begin{bmatrix} r_{ij}\cos(\theta_{ij}) \\ r_{ij}\sin(\theta_{ij}) \end{bmatrix}.$$
 (12)

Thus, a node needs only *one* neighbor that has localized itself to estimate its own position. Contrast this with the 3-connected requirement of r-only schemes that use geometric methods for estimating distances to anchors (Section 3.2). Furthermore, it follows the discussion in Section 3.2 that one would expect $r - \theta$ localization schemes to be effective even at densities of 5-6 nodes in the ranging neighborhood. We use simulation to verify if this is true.

But, in the presence of range and bearing measurement errors, this simple approach could propagate these errors throughout the network. How, then, can we accurately estimate position in a distributed fashion?

The approach we use (Least-Mean-Squares refi nement using Range and Bearing, or LMSRB) is very similar to the iterative refi nement scheme described for r-only localization in Section 3.1. Each $r - \theta$ measurement leads to the equation,

$$x_i - x_j - \Delta x_{ij} = 0, (13)$$

here Δx_{ij} is given by the second term on the right hand side of Equation 12.

As with LMSR, LMSRB fi nds the locations which 'best' fit the set of all readings. The basic idea is to cast the position estimation problem within an optimization framework, and then observe that minimizing the objective function is amenable to re-formulation in a manner that would permit distributed, iterative, computation (see Appendix A). The key difference here is that, because of the particular form of the objective function, there is one and only one solution. This means that, unlike LMSR, LMSRB does not require a 'good' initial guess.

LMSRB also weights $r-\theta$ measurements based on their level of certainty. For this we assume the $r-\theta$ measurements to have zero mean Gaussian error. However, we expect that our algorithms will not be significantly perturbed by any slight deviation from the Gaussian, since the Gaussian assumption merely provides a weighting mechanism. Let the standard deviations in measurements, r_{ij} and θ_{ij} be $\sigma_{r_{ij}}$ and $\sigma_{\theta_{ij}}$ respectively. Note, that each measurement could potentially have a different standard deviation. This allows for the use of sensors with different characteristics and variation of standard deviation with range. Assuming relatively small errors (up to about 10° cones in bearing errors) and ignoring third and higher order moments, Δx_{ij} can be approximated by a Gaussian random vector with covariance matrix:

$$\Delta M_{ij} = \begin{bmatrix} \sigma_{r_{ij}}^2 \cos^2(\theta_{ij}) + \sigma_{\theta_{ij}}^2 r_{ij}^2 \sin^2(\theta_{ij}) & \cos(\theta_{ij}) \sin(\theta_{ij}) \left(\sigma_{r_{ij}}^2 + r_{ij}^2 \sigma_{\theta_{ij}}^2\right) \\ \cos(\theta_{ij}) \sin(\theta_{ij}) \left(\sigma_r^2 + r_{ij}^2 \sigma_{\theta_{ij}}^2\right) & \sigma_{r_{ij}}^2 \sin^2(\theta_{ij}) + \sigma_{\theta_{ij}}^2 r_{ij}^2 \cos^2(\theta_{ij}) \end{bmatrix}.$$

$$(14)$$

One can use the covariance matrix (see Appendix A) as a weighting function to formulate the localization problem as a weighted least-squares optimization.

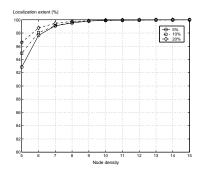
It turns out that the conditions for local minima in this formulation can be written as:

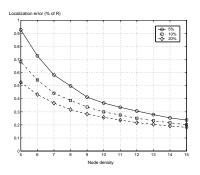
$$x_{i} = \left(\sum_{n_{i} \in A_{i}} \left(\Delta M_{ij}^{-1} \left(x_{j} + \Delta x_{ij} \right) + \Delta M_{ji}^{-1} \left(x_{j} - \Delta x_{ji} \right) \right) \right) \left(\sum_{n_{i} \in A_{i}} \Delta M_{ij}^{-1} + \Delta M_{ji}^{-1} \right)^{-1}, \forall i, j | n_{i} \in N - L, n_{j} \in N.$$
 (15)

In this equation, x_i depends only on the measured ranges and angles from its neighbors. Solving these equations using the Gauss Seidel iterative technique corresponds to solving the equations from an initial guess successively.

Thus, LMSRB would simply involve each node broadcasting its current position estimate locally to its neighbors (together with its own estimates of its distances and bearings to them). Each node would then use Equation 15 to update its current estimate and repeat the process until the estimate converges.

Results We evaluated the $r - \theta$ scheme for different node densities, three different anchor densities (5, 10 and 20%) and three different error values (0.1%, 1% and 5%). We used the same error model for ranging errors as used in evaluating r-only schemes. We evaluated the $r - \theta$ scheme for three different kinds of bearing error: low (a 2° cone), moderate (a 4° cone), and high (an 8° cone). The low values were motivated by laser rangefi nders (which are known to provide millimeter level accuracies at 10m, within a 2° cone). The higher values were motivated by a sonar ranging device or a less expensive laser device for bearing.





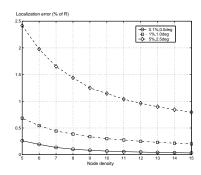


Figure 9: Localization Extent

Figure 10: Localization Error

Figure 11: Localization Error for Higher Bearing Error

We use the same simulation framework as for the r-only localization schemes (Section 3.3). The only new addition is the error model for bearing which we assume to be a zero mean Gaussian. A bearing measurement having a Gaussian error with standard deviation σ_{θ} , results in a cone of $4\sigma_{\theta}$. LMSRB scheme usually stabilizes within 20-30 iterations, depending on anchor ratios; however, we ran the experiments to 100 iterations to ensure asymptotic behavior. All the results are averaged over 50 simulations and the 99% confi dence interval for localization extent was 0.5% and for localization error 5cm.

As seen from Figure 9, even with an anchor fraction of 5% and node density of 5 the localization extent is more than 90% and reaches 98% by node density 6 for moderate error conditions. This follows from (and validates) the analytical discussion in Section 3.2. Furthermore, localization extent only improves with higher anchor fractions and node densities. Figure 10 depicts the localization error as a function of density at moderate error levels (4° cone). Even at an anchor fraction of 5% and node density of 5, mean localization error is less than 1% of ranging neighborhood (10cm in this case). As expected, performance only improves with increase in node densities and higher anchor fractions.

Figure 11 captures how errors in range and bearing effect the algorithm at 10% anchor fraction. An error of 2.5° translates to an average position error of $\frac{2R}{3}\sin 2.5=30\mathrm{cm}(3\%)$, since the expected range of a node is $\frac{2R}{3}$ and an angular deviation of θ at this distance implies a physical deviation of $\frac{2R}{3}\sin\theta$. Morover, a σ_{θ} of 2.5° implies an 8° cone. Also, considering that error might accumulate over multiple hops, it actually extremely encouraging that at high error, and a node density of 5, the mean error stays below 2.5% (25cm). The explanation for this is that the LMSRB is a global minimization and takes into account all the measurements in the sensor network. We did not, however, find any significant difference in localization extent with much higher anchor densities.

In conclusion, LMSRB provides a high localization extent and low localization errors even at low node densities and low anchor fractions.

Discussion Clearly, then, LMSRB shows that it is possible to achieve very good localization performance even at low densities (where, as before, by density we mean the number of nodes within ranging radius). Our LMSRB scheme is a contribution to the literature; to our knowledge no other work has attempted to use range *and* bearing information to obtain location (although a recent piece of work [13] attempts to use only bearing information to estimate position and requires the 3-connectedness property to work).

The key question is: Is $r - \theta$ localization practical? That is, can accurate bearing estimation devices be built at the form and energy factors that sensor nodes require? We do not know the answer to this question. Certainly, at larger form-factors, laser range fi nders can also estimate bearing, but they are also power-hungry and expensive.

One possible way to estimate bearing cheaply would be to use a ring of charge-coupled devices (CCDs) that detects light at certain frequencies. A node can emit a light pulse, and the CCD array on a receiving node can estimate bearing to the transmitter using little energy. However, while this seems plausible to us, we have not investigated how such a device might actually be engineered.

5 Range-Sector Based Localization Systems

Given that the feasibility of building accurate bearing estimation devices is an open question, we ask: if nodes could estimate imprecise bearing to each other, how would the resulting ad-hoc localization scheme perform? More precisely, if we allowed nodes to have *sector* estimation devices (those that could place a neighbor within, say a 45° sector), would the performance of r-sector localization schemes be closer to $r - \theta$ scheme or r-only schemes?

We now describe an r-sector distributed ad-hoc localization scheme, and evaluate its performance.

Iterative Least-Mean-Square Refinement with Range and Sector LMSRB cannot be directly extended to perform r-sector localization, for two reasons. First, sectoring devices are more likely to exhibit uniform, rather than Gaussian (or "close" to Gaussian) errors. Second, the first order approximation that we used to derive the covariance matrix in Equation 14 does not hold since the higher order terms will have a significant contribution beyond cones of 10° .

Instead, since the only relatively precise information available is the range information, our LMSRS (Iterative Least-Mean-Squares refi nement with Range and Sector) algorithm essentially uses the LMSR algorithm described in Section 3.1. LMSR requires a good initial guess for the locations of the nodes, and for this we use the sector information. A node gets an initial guess using Equation 13 and the center of the sector as an estimate for bearing to the neighbor.

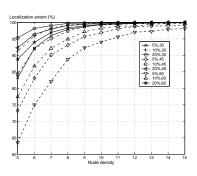
It is possible, with a little communication, to obtain a better initial guess; this is not crucial for LMSRS but can speed up its convergence. Consider Figure 14 where A has an initial position estimate, and node B is trying to get a position for itself using Equation 13. A and B announce to each other the sector that each thinks the other lies in (*e.g.*, B might say "A lies in my 45° to 90° sector"). Let CAD represents the sector which A thinks B is in. Let EBF be the sector which B thinks A is in. Suppose BF were the true bearing of A measured from B, then B would lie on the line AF'. Similarly if BE were the true bearing of A measured from B, then B would lie on somewhere on the line AE'. This means that using the sector information of B, B must lie in the sector F'AE'. The intersection of sectors CAD and F'AE' represents, a tighter bound on the region where B lies relative to A. Hence, now, B can initialize itself along the angle bisector of F'AD (AB') at a range $\frac{r_{AB}\sigma_{AB}^{-1} + r_{BA}\sigma_{BA}^{-1}}{\sigma_{AB}^{-1} + \sigma_{BA}^{-1}}$, as depicted in Figure 14. Here, r_{AB} and r_{BA} (and respectively the σ s) are ranges and standard deviations measured at A to B and vice versa. This approach provides a better initial guess only if the orientations of A and B are *not* identical, which is highly likely even in loosely planned deployments.

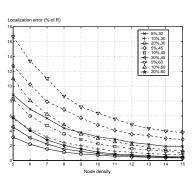
Our simulation setup for LMSRS is identical to that for LMSRB. In general, LMSRS takes about 40-50 iterations (recall that both LMSRB and LMSRS schemes are *distributed* schemes; when we say iterations we mean the number of message exchanges required to converge). All our simulations actually run over 100 iterations and reflect asymptotic results. Our results are averaged over 50 simulations to ensure that the 99% confi dence intervals for localization extent are less than 1% and for localization error are less than 5cm.

Results In our simulations, we found that that sensitivity of LMSRS to ranging error is relatively small at all anchor fractions and node densities. Accordingly (given the limited space), we discuss the how LMSRS performs for three different sector angles (30, 45 and 60) at three different anchor ratios (5%, 10% and 20%). In Figure 12 and Figure 13 we plot the variation of localization extent and error for these combinations. The most notable results are that at even as large 30° sectors, 10% anchor ratio and a density of 5 nodes, more than 90% nodes are localized within 2mts and the mean error is around 5.5% (55cm). The corresponding values at 20% anchor ratio are 95% and 3% (30cm). The performance improves at higher node densities and is expected to increase at higher anchor ratios. While these are extremely encouraging results, we examine the effect increased sector angle has on the performance. At 60° sectors and 20% anchor ratio and node density of 5, the localization extent is around 90% and the localization error is about 6% (60cm). This implies that even with sectors as large as 60° one could acheive 'satisfactory' localization if 20% anchors were deployed.

Discussion These results imply that an ad-hoc localization system that uses range and sector estimates can approach the performance offered by our $r - \theta$ scheme. More importantly, the localization extent and error are acceptable even at low densities.

As with $r - \theta$ localization, the question is: Are sector estimation devices really practical, particularly for sensor networks? We don't know, given that we know of no devices explicitly built to estimate sectors (directional antennas might, for example, be able to estimate sector bearing, but are not explicitly designed for that purpose). Of course, there hasn't been, till date, a pressing need to develop such a device. We believe our finding indicates that there is now





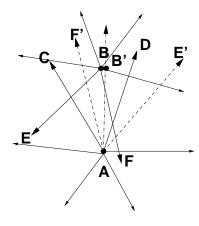


Figure 12: Localization extent for LMSRS Figure 13: Localization error for LMSRS

Figure 14: Obtaining an initial guess

a need; *r*-sector localization will enable ad-hoc localization systems at reasonable deployment densities, and ad-hoc localization seems to be an essential component of wireless sensor networks.

Certainly, it is more reasonable to expect that sector estimation devices will be easier to engineer, and perhaps cheaper, than bearing estimation devices. As before, a simple CCD array might actually be quite feasible, and we intend to examine the likelihood of manufacturing such a device.

6 Conclusions and Future Work

This paper makes the argument that the class of ad-hoc localization schemes considered in the literature so far fundamentally require high node densities in order to get acceptable performance. By contrast, new schemes that can use range and bearing estimates, or even range and sector estimates, can give good performance even at densities that ensure node connectivity.

Clearly much work is needed before ad-hoc localization systems that use ranging and sectoring devices become a reality. Sector estimation devices have to be built, our algorithms have to be validated by implementation and actual deployment in realistic environments. We are continuing to pursue these directions.

References

- [1] M. Mataric A. Howard and G. S. Sukhatme. Localization for Mobile Robot Teams Using Maximum Likelihood Estimation. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, EPFL, Switzerland, 2002.
- [2] P. Bahl and V.N. Padhmanabhan. RADAR An In-Building RF-Based User Location and Tracking System. In *Proc. of IEEE Infocom*, Tel Aviv, Isreal, 2000.
- [3] Nirupama Bulusu, J. Heidemann, and D. Estrin. GPS-less Low Cost Outdoor Localization for Very Small Devices. *IEEE Personal communications magazine, Special Issue on Smart Spaces and Environments*, 7(5), 2000.
- [4] S. Capkun, Maher Hamdi, and J. P. Hubaux. GPS-Free Positioning in Mobile Ad-Hoc Networks. *Cluster Computing*, 5(2), April 2002.
- [5] L. Doherty, K. Pister, and L. El-Ghaoui. Convex Position Estimation in Wireless Sensor Networks. In *Proceedings of IEEE Infocom*, 2001.
- [6] P. Enge and P. Misra. Scanning the Issue/Technology. *Proceedings of IEEE Special Issue on Global Positioning System*, 87(1), 1999.
- [7] D. Fox, W. Burgard, H. Kruppa, and S. Thrun. A Probabilistic Approach to Collaborative Multi-robot Localization. Autonomous Robots, 8(3):325–344, 2000.
- [8] L. Girod and D. Estrin. Robust Range Estimation Using Acoustic and Multimodal Sensing. In *Proc. IEEE International Conference on Intelligent Robots and Systems*, 2001.
- [9] A. Harter, A. Hopper, P. Steggles, A. Ward, and P. Webster. The Anatomy of a Context Aware Application. In *Proc. of ACM International Conference on Mobile Computing and Networking (MOBICOM)*, Seattle, WA, USA, 1999.

- [10] J. Hightower, C. Vakili, G. Borriello, and R. Want. Design and Calibration of the SpotON Ad-Hoc Location Sensing System, 2001.
- [11] K. Langendoen and N. Reijers. Distributed Localization in Wireless Sensor Networks: A Quantitative Comparison. Technical Report PDS-2002-003, Technical University, Delft, November 2002.
- [12] D. Niculescu and B. Nath. Ad-hoc Positioning System. In Proceedings of IEEE Globecom, 2001.
- [13] D. Niculescu and B. Nath. Ad-hoc Positioning System (APS) Using AOA. In *Proceedings of IEEE Infocom*, San Francisco, CA, 2003.
- [14] Nissanka B Priyantha, Anit Chakraborty, and Hari Balakrishnan. The Cricket Locations Support System. In Proc. of Sixth ACM International Conference on Mobile Computing and Networking (MOBICOM), Boston, Massachusetts, USA, 2000.
- [15] Nissanka B Priyantha, Alen K. L. Miu, Hari Balakrishnan, and Seth Teller. The Cricket Compass for Context-Aware Mobile Applications. In *Proc. of Sixth ACM International Conference on Mobile Computing and Networking (MOBICOM)*, Rome, Italy, 2001.
- [16] S. Roumeliotis and G. Bekey. Collective Localization: a Distributed Kalman Filter Approach. In *Proc. IEEE ICRA*, volume 3, pages 2958–2965, San Francisco, May 1999.
- [17] C. Savarese, K. Langendoen, and J. Rabaey. Robust Positioning Algorithms for Distributed Ad-Hoc Wireless Sensor Networks. In *Proc. Usenix Annual Technical Conference*, Monterey, CA, June 2002.
- [18] A. Savvides, W. Garber, S. Adlakha, R. Moses, and M. Srivastava. On the Error Characteristics of Multihop Node Localization in Wireless Sensor Networks. In *Proceedings of First International Workshop on Information Processing in Sensor Networks*, 2003.
- [19] A. Savvides, H. Park, and M. Srivastava. The Bits and Flops of the N-hop Multilateration Primitive for Node Localization Problems. In Proceedings of the First International Workshop for Wireless Sensor Networks and Applications (WSNA), 2002.
- [20] Andreas Savvides, Chih-Chien Han, and Mani Srivastava. Dynamic Fine-Grained Localization in Ad-Hoc Networks of Sensors. In ACM/IEEE International Conference on Mobile Computing and Networking, Rome, Italy, July 2001. ACM.
- [21] K. Whitehouse and D. Culler. Calibration as a Parameter Estimation Problem in Sensor Network. In Proc. ACM Workshop on Sensor Networks and Applications, Atlanta, GA, 2002.

A The Theory Behind our $r - \theta$ Localization Scheme

Continuing with the terminology defined in Section 4, our approach seeks to minimize the objective function,

$$J = \sum_{e_{ij} \in E} \left(x_i - x_j - \Delta x_{ij} \right)^T \Delta M_{ij}^{-1} \left(x_i - x_j - \Delta x_{ij} \right). \tag{A-1}$$

The objective function in A-1 is a commonly used weighted least mean square formulation, based on the Mahalanobis norm. The conditions for local minima are given by the set of |N| - |L| simultaneous equations,

$$\frac{\partial J}{\partial x_i} = \sum_{\forall n_j \in A_i} \left(\Delta M_{ij}^{-1} \left(x_i - x_j - \Delta x_{ij} \right) + \Delta M_{ji}^{-1} \left(x_i - x_j - \Delta x_{ji} \right) \right) = 0 \forall i, j | n_i \in N - L, n_j \in N. \tag{A-2}$$

This gives a set of |N| - |L| simulataneous equations,

$$\left(\sum_{n_{j} \in A_{i}} \left(\Delta M_{ij}^{-1} + \Delta M_{ji}^{-1}\right)\right) x_{i} - \sum_{n_{j} \in A_{i}} \left(\left(\Delta M_{ij}^{-1} + \Delta M_{ji}^{-1}\right) x_{j}\right) = \sum_{n_{j} \in A_{i}} \left(\Delta M_{ij}^{-1} \Delta x_{ij} - \Delta M_{ji}^{-1} \Delta x_{ji}\right) \forall i, j | n_{i} \in \mathbb{N} - L, n_{j} \in \mathbb{N}. \quad (A-3)$$

All these equations can be considely written in a matrix form as BX = f and X can be found as $B^{-1}f$. However, B^{-1} cannot be calculated in a distributed manner, moreover the size of B can be o(|N|), making this computationally infeasible, since there can be potentially thousands or even millions of nodes in the sensor field. The key to finding a distributed solution lies in re-writing these equations as,

$$x_{i} = \left(\sum_{n_{j} \in A_{i}} \left(\Delta M_{ij}^{-1} \left(x_{j} + \Delta x_{ij}\right) + \Delta M_{ji}^{-1} \left(x_{j} - \Delta x_{ji}\right)\right)\right) \left(\sum_{n_{j} \in A_{i}} \Delta M_{ij}^{-1} + \Delta M_{ji}^{-1}\right)^{-1} \forall i, j | n_{i} \in N - L, n_{j} \in N.$$
(A-4)

Since, the objective function is quadratic, there can only be one unique solution to this equation. As iterations proceed the scheme converges to the globally unique solution. It can be shown that this scheme is guranteed to converge from any initial state. A formal proof of convergence is given in Appendix B.

Proof convergence for $r - \theta$ adhoc localization B

Claim I: A necessary and sufficient condition for convergence of Gauss Seidal on BX = f is $\rho(M) < 1$, where B = I - C. Here, $\rho(C)$ is the spectral norm of C.

Proof: Let X_{θ} be the initial guess, and let X^t be the estimate after the t^{th} iteration. Then,

$$X^{t} = \left(I + \sum_{i=1}^{i=t-1} C^{i}\right) f + C^{t} X_{0}.$$
(B-1)

As $t \to \infty$, $X^t \to (I - C)^{-1} f$ iff $\rho(C) < 1$,

since $I + \sum_{i=1}^{i=\infty} C^i = (I - C)^{-1}$. Hence proved.

$$C \text{ is a symmetric matrix of the form,} \begin{bmatrix} 0 & H_{21} & \cdots & H_{|N|-|L|,1} \\ H_{21} & 0 & \cdots & 0 \\ \cdots & \cdots & 0 & H_{|N|-|L|,|N|-|L|-1} \\ H_{|N|-|L|,1} & \cdots & H_{|N|-|L|,|N|-|L|-1} & 0 \end{bmatrix}.$$

Here 0 is 2×2 zero matrix and H_{ij} are 2×2

Claim II: $\rho(C) < 1$ if $\rho(\sum_{l=1}^{l=n} H_{il})$, $\rho(\sum_{l=1}^{l=n} H_{li}) \le 1$ and H_{ij} are all positive definite or $0 \ \forall n_i \in N-L$ and at least one H_{ij} is positive definite.

Proof:

$$\rho(C) = max \left(\frac{\|CX\|}{\|X\|} \right)$$
 (B-2)

$$= \max\left(\frac{X^TC^TCX}{X^TX}\right) \forall X \in \Re^{2(|N|-|L|)}$$
(B-3)

Let $K = C^T C$, and $K_{ij} = \sum_{l=1}^{l=n} H_{il} H_{lj}$.

$$\rho\left(K_{ij}\right) = \rho\left(\sum_{l=1}^{l=n} H_{il} H_{lj}\right) \le \rho\left(\sum_{l=1}^{l=n} H_{il} H_{lj} + \sum_{l_1=1}^{l_1=n} \sum_{l_2=1}^{l_2=n} H_{il_1} H_{l_2j}\right)$$
(B-4)

since multiplication two positive definite or (0 matrix) matrices results in a positive definite matrix or (0 matrix) and addition of positive definite matrices or (0 matrix) results in a positive definite matrix with a larger (or equal) spectral norm.

$$\rho\left(\sum_{l=1}^{n} H_{il} H_{lj} + \sum_{l_1=1}^{n} \sum_{l_2=1}^{n} H_{il_1} H_{l_2j}\right) = \rho\left(\sum_{l=1}^{l=n} H_{il} \sum_{l=1}^{l=n} H_{li}\right) \le \rho\left(\sum_{i=1}^{i=n} H_{il}\right) \rho\left(\sum_{l=1}^{i=n} H_{il}\right) \le 1$$
(B-5)

If at least one H_{ij} is positive definite then, at least one $\rho(K_{ij}) < 1$.

The alreast one
$$H_{ij}$$
 is positive definite then, atleast one $\rho(K_{ij}) < 1$.

Let $X = \begin{bmatrix} x_1 \\ \cdots \\ x_{|N|-|L|} \end{bmatrix}$, $x_i \in \Re^2$.

$$X^T C^T C X = X^T K X = \sum \sum x_i^T K_{ij} x_j = \sum \sum \|x_i^T K_{ij} x_j\| \leq \sum \left(\|x_i^T x_j^T\| \|K_{ij}\| \right)$$

$$< \sum \sum \|x_i^T x_j^T\| = \sum \sum x_i^T x_j^T = X^T X.$$

Hence, $\frac{X^T C^T C X}{X^T X} < 1$, i.e. $\rho(C) < 1$.

Claim III: H_{ij} are all positive definite or 0.

Proof: Let *L* be the set of all anchors. If $n_i \in L$ then $H_{ij} = 0$, else

$$H_{ij} = \left(\sum_{j \in Adj(i)} \Delta M_{ij}^{-1} + \Delta M_{ij}^{-1}\right)^{-1} \left(\Delta M_{ij}^{-1} + \Delta M_{ji}^{-1}\right). \tag{B-6}$$

Now, ΔM_{ij} are all covariance matrices for gaussian random variables, and hence are positive definite. Sum, inverse and product of positive definite matrices are positive definite. Hence, H_{ij} are all positive definite.

Claim IV: $\rho\left(\sum_{l=1}^{l=n} H_{il}\right), \rho\left(\sum_{l=1}^{l=n} H_{li}\right) < 1.$

$$\sum_{l} H_{il} = \left(\sum_{n_j \in A_i} \Delta M_{ij}^{-1} + \delta M_{ji}^{-1}\right)^{-1} \sum_{j \mid n_j \notin L} \Delta M_{ij}^{-1} + \Delta M_{ji}^{-1}.$$
(B-7)

Let $\sum_{n_j \notin L} \Delta M_{ij}^{-1} + \Delta M_{ji}^{-1} = a_i$ and $\sum_{n_j \in L} \Delta M_{ij}^{-1} + \Delta M_{ji}^{-1} = b_i$, then $\sum_{l=1}^{l=n} H_{il} = (a_i + b_i)^{-1} a_i$. Since, ΔM_{ij} are positive definite symmetric matrices, so are a_i and b_i . This means there exists a rotated coordinate system (unitary transform) which diagonalizes a_i without altering its eigen values. Let $a_i = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ and $b_i = \begin{bmatrix} \alpha & \gamma \\ \gamma & \beta \end{bmatrix}$. Here, $\alpha\beta > \gamma^2$ and $\lambda_1, \lambda_2 > 0$.

$$(a_i+b_i)^{-1}a_i = \frac{1}{(\alpha+\lambda_1)\,(\beta+\lambda_2)-\gamma^2} \left[\begin{array}{cc} (\beta+\lambda_2)\,\lambda_1 & -\gamma\lambda_2 \\ -\gamma\lambda_1 & (\alpha+\lambda_1)\,\lambda_2 \end{array} \right]. \tag{B-8}$$
 The largest eigen value than is given by,
$$\frac{(2\lambda_1\lambda_2+\alpha\lambda_2+\beta\lambda_1)+\sqrt{(2\lambda_1\lambda_2+\alpha\lambda_2+\beta\lambda_1)^2-4\lambda_1\lambda_2((\alpha+\lambda_1)(\beta+\lambda_2)-\gamma^2)}}{2((\alpha+\lambda_1)(\beta+\lambda_2)-\gamma^2)}.$$
 It is then straight forward to that the eigen value is less than 1 iff $0<\alpha\beta-\gamma^2$, which is true since b_i is positive definite. The same argument can be used to prove that $\alpha(\Sigma H_1)<1$. Hence the claim is proved.

The same argument can be used to prove that $\rho(\Sigma H_{li}) < 1$. Hence the claim is proved.

Claims I,II,III and IV when combined imply that, the algorithm is guaranteed to converge monotonically to a globally unique solution.

Note that, if not even one H_{ij} is non-zero, it means that, all nodes in the network are isolated i.e. have no neighbor and the localization is impossible.