Bilingual Mathematics Learners: How Views of Language, Bilingual Learners, and Mathematical Communication Impact Instruction

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Understanding the relationship between language and mathematics learning is crucial to designing mathematics instruction for students who are English Learners (ELs) and/or bilingual.[1] Before we can address questions about instruction for this population, we need to first examine views of bilingual mathematics learners and how they use language to communicate mathematically. This chapter considers how our conceptions of bilingual mathematics learners impact instruction for this population. In particular, I examine how views of the relationship between mathematics and language constrain instruction. I describe three views of bilingual mathematics learners, examine how these views impact instruction, and critique these views using a sociocultural perspective.

Understanding bilingual mathematics learners and developing principled instruction is a pressing practical issue, particularly for Latino students. An increasing number of school age children in the U.S. are Latinos; Latino students constitute the majority many major urban school districts in the country (Young, 2002). By 2050 there will be approximately 100 million Latinos in the U.S. In the future, most public school teachers in cities, suburbs and rural areas will be teaching Latino children. Mathematics achievement scores for Latinos on tests such as NAEP (National Assessment of Educational Progress) fall below Anglo-American and African-American students and the gap between 1990 and 2003 NAEP scores for Whites and Latinos did not change significantly (NCES, 2004). These are all good reasons to examine how views of bilingual mathematics learners (Latinos in particular) impact instruction.

Early studies of bilingual students learning mathematics focused on word problems, especially translating word problems from English to mathematical symbols. Most of these studies characterized the challenges that bilingual students faced as acquiring vocabulary or struggling with the mathematics register. Recommendations for instruction for English learners that emphasize vocabulary and reading comprehension skills reflect this focus. In contrast, current research on mathematics learning emphasizes how students construct multiple meanings, negotiate meanings through interactions with peers and teachers, and participate in mathematical communication. Although research has explored mathematical communication as a central aspect of learning mathematics in monolingual classrooms, few studies have addressed mathematical communication in bilingual classrooms (for examples see Adler, 1998; Brenner, 1994; Khisty, McLeod, & Bertilson, 1990; Khisty, 1995; and Moschkovich, 1999).

The increased emphasis on mathematical communication in reform classrooms could result in several scenarios. On the one hand, this emphasis could create additional obstacles for bilingual learners. On the other hand, it might provide additional opportunities for bilingual learners to flourish. And lastly, it might create a combination of these two scenarios, depending on the classroom context. Without empirical studies that explore these hypothetical scenarios and examine mathematical communication in classrooms with bilingual students, it is impossible to reach conclusions regarding the impact of reform on bilingual learners. When carrying out these studies or designing instruction, we need to first consider how we conceptualize language, bilingual learners, and mathematical communication. As researchers, designers, or teachers we can only see what our conceptual frameworks allow us to see. Our views will have great impact on our conclusions and recommendations.

The aim of this chapter is to describe three views of bilingual mathematics learners and explore how these views impact instruction and equity for this population. I examine three perspectives on bilingual mathematics learners, describe how the first two constrain research and instruction, and consider how a sociocultural perspective can inform our understanding of the processes underlying learning mathematics when learning English. The first perspective emphasizes acquiring vocabulary, the second emphasizes multiple meanings, and the third emphasizes participation in mathematical Discourse practices. The third perspective is a situated and sociocultural[2] view of language and mathematics learning that uses the concepts of registers (Halliday, 1978) and Discourses (Gee, 1996 and 1999).

I question the efficacy of the first two perspectives for understanding bilingual mathematics learners and designing instruction for this population. The first two views can create inequities in the classroom because they emphasize what learners don't know or can't do. In contrast, a sociocultural perspective shifts away from deficiency models of bilingual learners and instead focuses on describing the resources bilingual students use to communicate mathematically. Without this shift we will have a limited view of these learners and we will design instruction that neglects the competencies they bring to mathematics classrooms. If all we see are students who don't speak English, mispronounce English words, or don't know vocabulary, instruction will focus on these deficiencies. If, instead, we learn to recognize the mathematical ideas these students express in spite of their accents, code-switching, or missing vocabulary, then instruction can build on students' competencies and resources.

Below I describe three perspectives of bilingual mathematics learners: acquiring vocabulary, constructing multiple meanings, and participating in Discourse practices.[3] I argue that the third view, a sociocultural perspective, enriches our views of the relationship between language and learning mathematics, expands what counts as competence in mathematical communication, and provides a basis for designing equitable instruction. To make this case, I first compare and contrast the three perspectives and then present two examples to substantiate my claims regarding the contributions of a sociocultural perspective.

Acquiring Vocabulary

One view of bilingual mathematics learners is that their main challenge is acquiring vocabulary. This first perspective defines learning mathematics as learning to carry out computations or solve traditional word problems, and emphasizes vocabulary as the central issue for English learners as they learn mathematics. This view is reflected in early research on bilingual mathematics learners that focused primarily on how students understood individual vocabulary terms or translated traditional word problems from English to mathematical symbols (for examples, see Cuevas, 1983; Cuevas, Mann, & McClung, 1986; Mestre, 1981 and 1988; Spanos & Crandall, 1990; Spanos, Rhodes, Dale, & Crandall, 1988). Recommendations for mathematics instruction for English learners have also emphasized vocabulary and reading comprehension (Dale & Cuevas, 1987; MacGregor & Moore, 1992; Olivares, 1996; Rubenstein, 1996).

Although an emphasis on vocabulary and reading comprehension may have been sufficient in the past, this emphasis does not match current views of mathematical proficiency or the activities in contemporary classrooms. In many mathematics classrooms today, the main activities are not carrying out arithmetic computations, solving traditional word problems, reading textbooks, or completing worksheets. Many students participate in a variety of oral and written practices such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments. As a consequence, reading and understanding mathematical texts or traditional word problems are no longer the best examples of how language and learning mathematics intersect.

Even in traditional classrooms where there may be little oral discussion, learning mathematical language involves more than learning vocabulary: words have multiple meanings,

meanings depend on situations, and learning to use mathematical language requires learning when to use different meanings. Vocabulary (along with decoding) is certainly an aspect of developing reading comprehension *at the word level*. However, vocabulary is not sufficient for becoming a competent reader. Reading comprehension involves skills beyond the word level, such as constructing meaning from text, using metacognitive strategies, and participating in academic language practices (Pressley, 2000).

An emphasis on vocabulary provides a narrow view of mathematical communication. This narrow view can have a negative impact on assessment and instruction for bilingual learners. English oral proficiency can affect how teachers assess a student’s mathematical competence. For example, if we focus only on a student's failure to use the correct word, we can miss the student’s competency in making conjectures, constructing arguments, addressing special cases, or dealing with contradictory evidence. If we conceive of “language” as only vocabulary, we are limiting the scope of communicative activities used to assess mathematical competence, and many students will appear less competent. Instruction focusing on low-level linguistic skills, such as vocabulary, neglects the more complex language skills necessary for learning and doing mathematics.

Lastly, this view perpetuates a deficiency model of bilingual learners that can have a negative impact on English learners’ access to mathematical instruction. English learners may have a smaller or less accurate mathematical vocabulary in English than native English speakers. We can see this as a deficiency or we can notice this difference while also noticing other competencies for communicating mathematically. “Vocabulary” need not be construed as a deficiency, a reason for remedial instruction, or a pre-requisite that bilingual learners must achieve before they can participate in more conceptual or advanced mathematics instruction.

English learners can learn vocabulary at the same time they participate in many types of lessons, including conceptual mathematical activities.

Constructing Multiple Meanings

A second perspective on bilingual mathematics learners describes learning mathematics as constructing multiple meanings for words. Work in mathematics education from this perspective has used the notion of the mathematics register. Halliday (1978) defined register in the following way:

> A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the "mathematics register," in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

A register is a language variety associated with a particular situation of use. Common examples of registers include legal talk and baby talk. The notion of register includes not only lexical items but also phonology, morphology, syntax, and semantics as well as non-linguistic behavior. The notion of register thus involves aspects of the situation[4]. From this perspective, since there are multiple meanings for the same term, students who are learning mathematics are learning to use these different meanings appropriately in different situations. There are several examples of such multiple meanings: the phrase "any number" means "all numbers" in a math context (Pimm, 1987); "a quarter" can refer to a coin or to a fourth of a whole (Khisty, 1995); and in Spanish "un cuarto" can mean a room or a fourth (Khisty, 1995).

Multiple meanings can create obstacles in mathematical conversations because students often use colloquial meanings while the teacher (or other students) may use mathematical meanings. For example, the word “prime” can have different meanings depending on whether it is used to refer to “prime number,” “prime time,” or “prime rib.” In Spanish “primo” can mean “cousin” or “prime number” as in the phrase “número primo.” Another example of multiple meanings is Walkerdine’s (1998) description of the differences between the meanings of "more" in the mathematics classroom and at home. While in a classroom situation “more” is usually understood to be the opposite of “less,” at home the opposite of “more” is usually associated with “no more” as in, for example, “I want more paper” and “There is no more paper.”

The multiple meanings perspective considers differences between the everyday and mathematical registers. This perspective has contributed to descriptions of how learning mathematics involves, in part, a shift from everyday to more mathematical and precise meanings. For example, studies have described how students’ language use moves closer to the mathematics register by becoming more precise and reflecting deeper conceptual knowledge (Moschkovich, 1996, 1998; O’Connor, 1992).

Using two national languages, for example English and Spanish, may complicate moving across two registers. For example, distinguishing between the two uses of “más”[*more*] below is crucial in a mathematics context:

hay cuatro más ___ que ___ [*there are four more* ___ *than* ___]

hay cuatro veces más ___ que ___ [*there are four times as many* ___ *as* ____]

These two sentences refer to two different mathematical situations and yet the word “más” [*more*] is used in both cases.

The multiple meanings perspective adds complexity to our view of the relationship between language and learning mathematics. Emphasizing multiple meanings shifts the focus from examining how students acquire vocabulary to examining how students negotiate the multiple meanings of mathematical terms, from acquiring words to developing meanings for those words, from learning words with single meanings to understanding multiple meanings, and from learning vocabulary to using language appropriately in different situations.

This perspective should not be interpreted to imply that the two registers are separate or that everyday meanings are necessarily obstacles. Forman (1996) offers evidence that the two registers do not function separately, but that students and teacher interweave the everyday and academic registers in classroom discussions. Although differences between the everyday and mathematical registers are sometimes obstacles for communicating in mathematically precise ways and everyday meanings can sometimes be ambiguous, everyday meanings are not always obstacles. Everyday metaphors, meanings, and experiences can also provide resources for understanding mathematical concepts. For example, Moschkovich (1996) has described how students used a metaphor drawing on everyday experiences (describing a steeper line as harder to climb than a line that is less steep) to compare the steepness of lines on a graph.

The two perspectives summarized above, “acquiring vocabulary” and “constructing multiple meanings,” have provided useful analytical tools. However, they can be used in ways that have negative implications for equity in classrooms. If these perspectives are used to emphasize the *obstacles* that bilingual students face as they learn mathematics, they provide only deficiency models (Garcia & Gonzalez, 1995; Gonzalez, 1995) of bilingual students as mathematics learners. Instead of emphasizing obstacles, we need to consider the resources bilingual learners use for learning mathematics. In the next section I explore how a sociocultural view can provide a more complex view of bilingual mathematics learners and shift the emphasis from deficiencies and obstacles to resources and competencies.

Participating in Mathematical Discourse Practices

The sociocultural perspective described here uses a situated perspective of learning mathematics (Brown, Collins, & Duguid, 1989; Greeno, 1994) and the notion of Discourses (Gee, 1996) to build on previous work on classroom mathematical and scientific discourse (Cobb, et al., 1993; Rosebery et al., 1992). This perspective implies, first, that learning mathematics is viewed as a discursive activity (Forman, 1996). From this perspective, learning mathematics involves participating in a community of practice (Forman, 1996; Lave and Wenger, 1991; Nasir, 2002), contributing to the development of classroom socio-mathematical norms (Cobb et al., 1993), and using multiple material, linguistic, and social resources (Greeno, 1994). This perspective assumes that learning is inherently social and cultural "whether or not it occurs in an overtly social context" (Forman, 1996, page 117), that participants bring multiple views to a situation, that words, representations, and inscriptions have multiple meanings, and that participants actively negotiate these multiple meanings.

Rather than defining a "bilingual learner" as an individual who is proficient in more than one language, a sociocultural perspective defines bilingual learners as students who participate in multiple language communities. As described by Valdés-Fallis (1978), "natural" bilinguals are "the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations" (p. 4). Work in sociolinguistics has described code switching as one of the resources available to bilingual speakers. These studies have shown that code switching is a rule- and constraint-governed process and a dynamic verbal strategy in its own right, rather than a sign that students are deficient or "semi-lingual." This work also cautions that code switching should not be seen as a deficiency or a reflection of the ability to recall (Valdés-Fallis, 1978).

A sociocultural perspective views language as more than sequential speech or writing. Gee emphasizes how "Discourses always involve more than language " (1999, page 25) and defines Discourses as much more than vocabulary or multiple meanings:

> A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role. (Gee 1996, p. 131)

Using Gee's definition, mathematical Discourses include not only ways of talking, acting, interacting, thinking, believing, reading, and writing but also communities, values, beliefs, points of view, objects, and gestures.

There is no one mathematical Discourse or practice (for a discussion of multiple mathematical Discourses see Moschkovich, [2002a]). Mathematical Discourses involve different communities (mathematicians, teachers, or students) and different genres (explanations, proofs, or presentations). Practices vary across communities of research mathematicians, traditional classrooms, and reform classrooms. However, within these various communities, there are commonalities in the practices that count as participation in competent mathematical Discourse. Particular modes of argument, such as precision, brevity, and logical coherence, are valued (Forman, 1996). In general, abstracting, generalizing, searching for certainty, and being precise, explicit, brief, and logical are highly valued activities across different mathematical communities. Mathematical claims apply only to a precisely and explicitly defined set of situations, as in the statement "multiplication makes a number bigger, except when multiplying by a number smaller than 1." Claims are frequently tied to mathematical representations such as graphs, tables, or diagrams. The value of generalizing is reflected in common mathematical statements, such as "the angles of any triangle add up to 180 degrees," "parallel lines never meet," or "a + b will always equal b + a." Imagining (for example, infinity or zero), visualizing, hypothesizing, and predicting are also valued mathematical practices.

Mathematical Discussions

In this section I examine two mathematical discussions to illustrate the limitations of the vocabulary and multiple meanings perspectives and to describe how a sociocultural perspective enriches our view of language, provides an alternative to deficiency models of learners, and generates different questions for both reseacrh and instruction. I selected the first example to illustrate the limitations of the vocabulary perspective and the second example to illustrate the limitations of the multiple meanings perspective. The two examples presented below show the complexity that using a situated and sociocultural perspective as an analytical lens brings to the study of bilingual mathematics learners. The first example shows us how the vocabulary perspective fails to capture students' competencies in communicating mathematically. The second example shows that the multiple meanings perspective can also fall short of a full description of the resources that students use.

In presenting these examples, I also show how to use a sociocultural perspective to identify student competencies and resources that instruction can build on to support mathematics learning. To uncover these competencies and resources, I use the following questions, selectively and loosely following Gee's (1999) questions for Discourse analysis: 1) What are the situated meanings of the words and phrases that seem important in the situation? 2) What are the resources students use to communicate mathematically? What sign systems (speech, writing, images, and gestures) are relevant in the situation? In particular, how is "stuff" other than language relevant? And 3) What Discourses are involved? What Discourse practices are students participating in that are relevant in mathematical communities or that reflect mathematical competence?

Example 1: Describing a Pattern

A group of seventh and eighth grade students in a summer mathematics course constructed rectangles with the same area but different perimeters and looked for a pattern to relate the dimensions and the perimeter of their rectangles. Below is a problem similar to the one they were working on:

1. Look for all the rectangles with area 36 and write down the dimensions.
2. Calculate the perimeter for each rectangle.
3. Describe a pattern relating the perimeter and the dimensions.

In this classroom, there was one bilingual teacher and one monolingual teacher. A group of four students were videotaped as they talked in their small group and with the bilingual teacher (primarily in Spanish). They attempted to describe the pattern in their group and searched for the Spanish word for rectangle. The students produced several suggestions, including ángulo [*angle*], triángulo [*triangle*], rángulos, and rangulos. Although these students attempted to find a term to refer to the rectangles neither the teacher nor the other students provided the correct Spanish word, rectángulo [*rectangle*].

Later on, a second teacher (monolingual English speaker) asked several questions from the front of the class. In response, one of the students in this small group, Alicia, described a relationship between the length of the sides of a rectangle and its perimeter. (Transcript annotations are between brackets. Translations are in italics).

1. Teacher B: [Speaking from the front of the class] Somebody describe what they saw as a comparison between what the picture looked like and what the perimeter was . . .
2. Alicia: The longer the ah, . . . the longer [traces the shape of a long rectangle with her hands several times] the ah, . . . the longer the, rángulo [*rangle*], you know the more the perimeter, the higher the perimeter is.

Insert Figure 1: Alicia Describing a Rectangle, Part 1

Insert Figure 2: Alicia Describing a Rectangle, Part 2

An analysis of this excerpt using the vocabulary perspective would focus on this student's failed attempt to use the right word, "rectangle." Focusing on how vocabulary was an obstacle would not do justice to how this student successfully communicated a mathematical idea. If we were to focus only on Alicia's inaccurate use of the term "rángulo,"[5] we might miss how she used resources from the situation and how her statement reflects valued mathematical Discourse practices. If we move from a focus on vocabulary, then we can begin to see this student's competence. Alicia's competence only becomes visible if we include gestures and objects as resources for communicating mathematically. This move is important for instruction because it shifts the focus from a perceived deficiency in the student that needs to be corrected (not using the word rectangle) to a competency that can be refined through instruction (using gestures and objects). This move also shifts our attention from words to mathematical ideas, as expressed not only through words but also other modes. This shift is particularly important to uncover the mathematical competencies for students who are learning English.

Alicia used gestures to illustrate what she meant, and she referred to the concrete objects in front of her, the drawings of rectangles, to clarify her description. Alicia also used her first language as a resource. She interjected an invented Spanish word into her statement. In this way, a gesture, objects in the situation, and the student's first language served as resources for describing a pattern. Even though the word that she used for rectangle does not exist in either Spanish or English, it is very clear from the situation that Alicia was referring to a rectangle. It is also clear from her gestures that even though she did not use the words “length” or “width,” she was referring to the length of the side of a rectangle parallel to the floor.

Using a sociocultural perspective we can also ask what mathematical Discourse practices are relevant to this situation. Describing patterns is a paradigmatic practice in mathematics, so much so that mathematics is often defined as “the science of patterns” (Devlin, 1998, p. 3). And Alicia certainly described a pattern correctly. The rectangle with area 36 that has the greatest perimeter (74) is the rectangle with the longest possible length, 36, and shortest possible width, 1. As the length gets longer, say in comparing a rectangle of length 12, width 3, and perimeter 30 with a rectangle of perimeter 74, the perimeter does in fact become greater. Alicia appropriately (in the right place, at the right time, and in the right way) used a construction commonly used in mathematical communities to describe patterns, make comparisons, and describe co-variation: “the longer the _____, the more (higher) the _______.”

This example illustrates how a sociocultural perspective can open the way for seeing competence. This perspective does not emphasize the obstacles Alicia faced, but uncovers the ways that she used resources from the situation to communicate mathematically. Focusing on mathematical Discourse practices and including gestures and objects as resources make her mathematical competence visible.

Different implications for instruction follow from the vocabulary and sociocultural perspectives. Certainly, Alicia needs to learn the word for rectangle (ideally in both English and Spanish) but instruction should not stop there. Rather than only correcting her use of "rángulo" or recommending that she learn vocabulary, instruction should also build on Alicia's use of gestures, objects, and description of a pattern. If instruction only focuses on what mathematical terminology English learners know or don't know, they will always seem deficient because they are, in fact, learning a second language. If teachers perceive these students as deficient and only correct their vocabulary use, there is no room for addressing their mathematical ideas, building on these ideas, and connecting these ideas to the discipline. English learners thus run the risk of getting caught in a repeated cycle of remedial instruction that does not focus on mathematical content. Seeing mathematical communication as more than vocabulary implies that instruction should also focus on how students generalize, abstract, and describe patterns, rather than only on how students use individual words.

Example 2: Clarifying a Description

While the first example fits the expectation that bilingual students struggle with vocabulary, the vocabulary perspective was not sufficient to describe that student's competence. The second example highlights the limitations of the vocabulary perspective for describing mathematical communication and shows how code switching can be a resource for bilingual speakers. In the following discussion two students used both languages not for vocabulary, but to clarify the mathematical meaning of a description.

The example is taken from an interview conducted with two ninth-grade students after school. The students had been in mainstream English-only mathematics classrooms for several years. One student, Marcela, had some previous mathematics instruction in Spanish. These two students were working on the problem in Figure 3.

Insert Figure 3: Problem for Example 2

They had graphed the line y = -0.6x on paper and were discussing whether this line was steeper or less steep than the line y = x.

Insert Figure 4: Lines Drawn by the Students

Giselda first proposed that the line was steeper, then less steep. Marcela repeatedly asked Giselda if she was sure. After Marcela proposed that the line was less steep, she proceeded to explain her reasoning to Giselda. (Transcript annotations are between brackets. Translations are in italics directly below the utterance in Spanish).

1. Marcela: No, it’s less steeper . . .
2. Giselda: Why?
3. Marcela: See, it’s closer to the x-axis . . . [looks at Giselda] . . . Isn’t it?
4. Giselda: Oh, so if it’s right here . . . it’s steeper, right?
5. Marcela: Porque fíjate, digamos que este es el suelo.

   [Because look, let’s say that this is the ground.]

Entonces, si se acerca más, pues es menos steep.

[Then, if it gets closer, then it's less steep.]

. . . . 'cause see this one [referring to the line y = x] . . . is . . .

está entre el medio de la x y de la y. Right?

[is between the x and the y]

6. Giselda: [Nods in agreement.]

7. Marcela: This one [referring to the line y=-0.6x] is closer to the x than to the y, so this one [referring to the line y=-0.6x] is less steep.

The vocabulary perspective is not very useful for understanding what this student knows, describing how she communicates mathematically, or guiding instruction. Marcela, rather than struggling with vocabulary or using Spanish to fill in for a missing English word, used her first language to clarify a mathematical description. Marcela's competence involved more than knowing the meaning of "steeper" and "less steep." If we use a multiple meanings perspective, we can begin to see that in this discussion the two students are negotiating and clarifying the meanings of "steeper" and "less steep." We could say that Marcela used the mathematics register as a resource to communicate mathematically. She used two constructions common in the school mathematics register, "let's say this is . . ." and "if ____, then ____."

The multiple meanings perspective is also not sufficient for describing Marcela's competence. This becomes apparent when we focus on how this student used her first language, code switching, mathematical artifacts—the graph, the line y = x, and the axes—, and everyday experiences as resources. The premise that meanings from everyday experiences are obstacles for communicating mathematically does not hold for this example. In fact, Marcela used her everyday experiences and the metaphor that the x-axis is the ground "Porque fíjate, digamos que este es el suelo" [*Because look, let's say that this is the ground*] as resources for explaining her description. Rather than finding everyday meanings as obstacles for moving between two registers, she used an everyday situation to clarify her explanation.

Using a sociocultural perspective we can also ask what mathematical Discourse practices are relevant to this situation. Marcela's explanations echo mathematical Discourse practices in several ways. First, Marcela explicitly stated an assumption, a discursive practice valued in mathematical Discourse, when she said: "Porque fíjate, digamos que este es el suelo" [*Because look, let's say that this is the ground*]. Second, she supported her claim by making a connection to mathematical representations, another valued discursive practice. She used the graph, in particular the line y = x (line 5) and the axes (lines 5 and 7), as references to support her claim about the steepness of the line. A sociocultural perspective helps us to see that Marcela was participating in two discursive practices that reflect important values, stating assumptions explicitly and connecting claims to mathematical representations.

Conclusions

The three perspectives I have described make different assumptions regarding bilingual learners, define mathematical communication in different ways, and result in different recommendations for instruction (see Figure 5).

Insert Figure 5: Comparing Assumptions of the Three Views

The first two perspectives have been important in understanding the relationship between learning mathematics and language. They have also provided a basis for designing instruction for bilingual mathematics learners. A perspective that emphasizes acquiring vocabulary has been used to describe how students solve word problems and understand mathematical texts, and suggests that instruction should focus on vocabulary. A perspective that emphasizes constructing multiple meanings across registers has uncovered possible sources of misunderstandings in classroom conversations. This second perspective suggests that instruction can support bilingual learners in communicating mathematically by clarifying multiple meanings, addressing the conflicts between two languages explicitly, and discussing the different meanings students may associate with mathematical terms in each language.

As I have illustrated, these two perspectives have limitations. A focus on vocabulary does not capture the complexity of mathematical communication, ignores situational resources, and neglects important aspects of student mathematical competence. Assuming that students' everyday experience is an obstacle for learning mathematics obscures how everyday meanings can be resources for mathematical discussions. When these two perspectives are used to emphasize obstacles, they provide a limited model of bilingual students as mathematics learners that focuses on deficiencies. A more complete description of mathematical communication for bilingual students should include not only an analysis of the difficulties these students face but also of the competencies and resources they use to communicate mathematically.

The sociocultural perspective that I have presented expands the analytical lens to include non-language resources and mathematical Discourse practices, thus expanding what counts as competent mathematical communication. The key assumptions on which this broader view of competence is based include:

- Mathematical communication involves more than language;
- Meanings are multiple, changing, situated, and sociocultural; and

- Bilingual learners may be different than monolinguals but they should not be defined by deficiencies.

The two examples illustrate several aspects of learning mathematics in a bilingual classroom that only become visible when using a sociocultural perspective:

1) Learning to participate in mathematical Discourse is not merely or primarily a matter of learning vocabulary. During conversations in mathematics classrooms students are also learning to participate in valued mathematical Discourse practices such as describing patterns, making generalizations, and using representations to support claims.
2) Bilingual learners use many resources to communicate mathematically: gestures, objects, everyday experiences, their first language, code switching, and mathematical representations.
3) There are multiple uses of Spanish in mathematical conversations between bilingual students. While some students use Spanish to label objects, other students use Spanish to explain a concept, justify an answer, or elaborate on an explanation or description.
4) Bilingual students bring multiple competencies to the classroom. For example, even a student who is missing vocabulary may be proficient in describing patterns, using mathematical constructions, or presenting mathematically sound arguments.

A sociocultural perspective points to several aspects of classroom instruction that need to be considered. Classroom instruction should support bilingual students' engagement in conversations about mathematics, going beyond translating vocabulary and involving students in communicating about mathematical ideas. The examples presented here show that English learners can participate in discussions in which they grapple with significant mathematical ideas, even when they do not use the right words or switch languages.

Instruction for this population should not emphasize low-level language skills over opportunities to actively and repeatedly communicate about mathematical ideas. One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical ideas, rather than on pronunciation, vocabulary, or low-level linguistic skills. By learning to recognize how bilingual students express their mathematical ideas as they are learning English, teachers can maintain a focus on the mathematical ideas as well as on language development.

It is not a question of whether or not students should learn vocabulary but rather how instruction can best support students learning both vocabulary and mathematics. Vocabulary drill and practice is not the most effective instructional practice for learning either vocabulary or mathematics. Instead, vocabulary and second language acquisition experts describe vocabulary acquisition in a first or second language as occurring most successfully in instructional contexts that are language rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000; Pressley, 2000). To develop written and oral communication skills students need to participate in negotiating meaning (Savignon, 1991) and in tasks that require output from students (Swain, 2001). In sum, instruction should provide opportunities for students to actively use mathematical language to communicate about and negotiate meaning for mathematical situations.

Understanding the mathematical ideas in what students say and do can be difficult when teaching, perhaps especially so when working with students who are learning English. It may not be easy (or even possible) to sort out which aspects of a student's utterance are results of the student's conceptual understanding or the student's English proficiency. However, if the goal of instruction is to support students as they learn mathematics, determining the origin of an error is not as important as listening for students' mathematical ideas and uncovering the mathematical competence in what they are saying and doing. Hearing mathematical ideas and uncovering mathematical competence is only possible if we move beyond limited views of language and deficiency models of bilingual learners.

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Figure 1: Alicia Describing a Rectangle, Part I

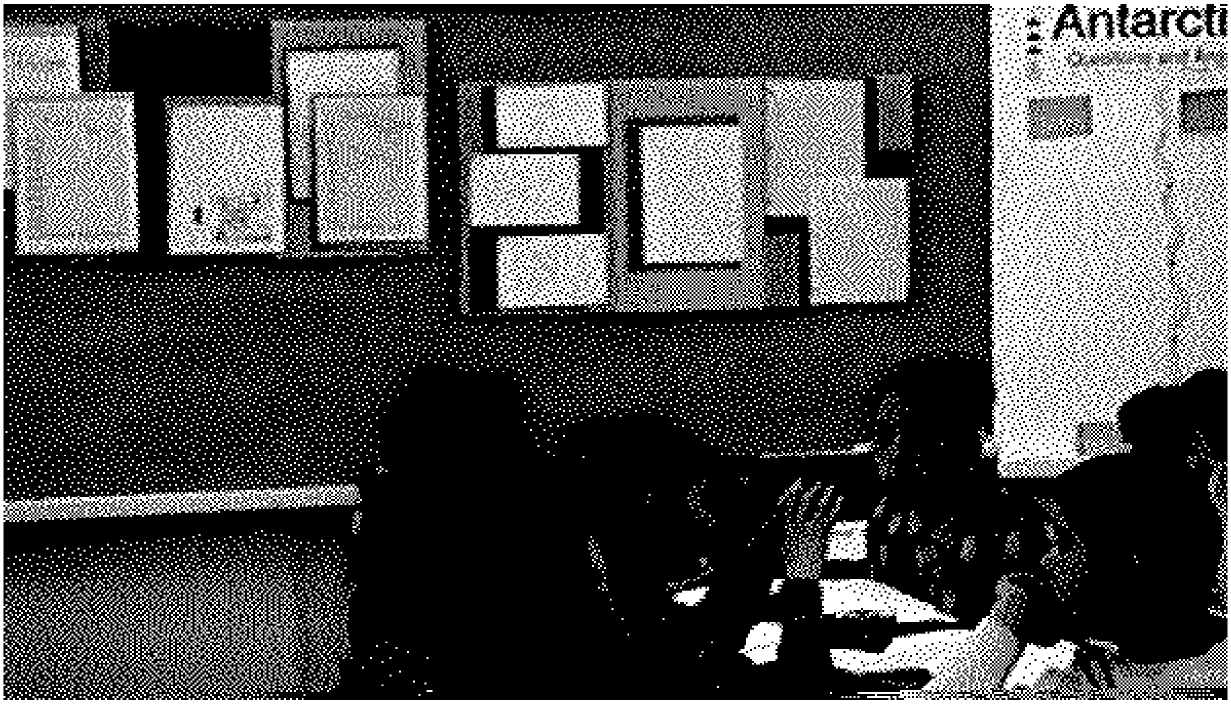


Figure 2: Alicia Describing a Rectangle, Part II

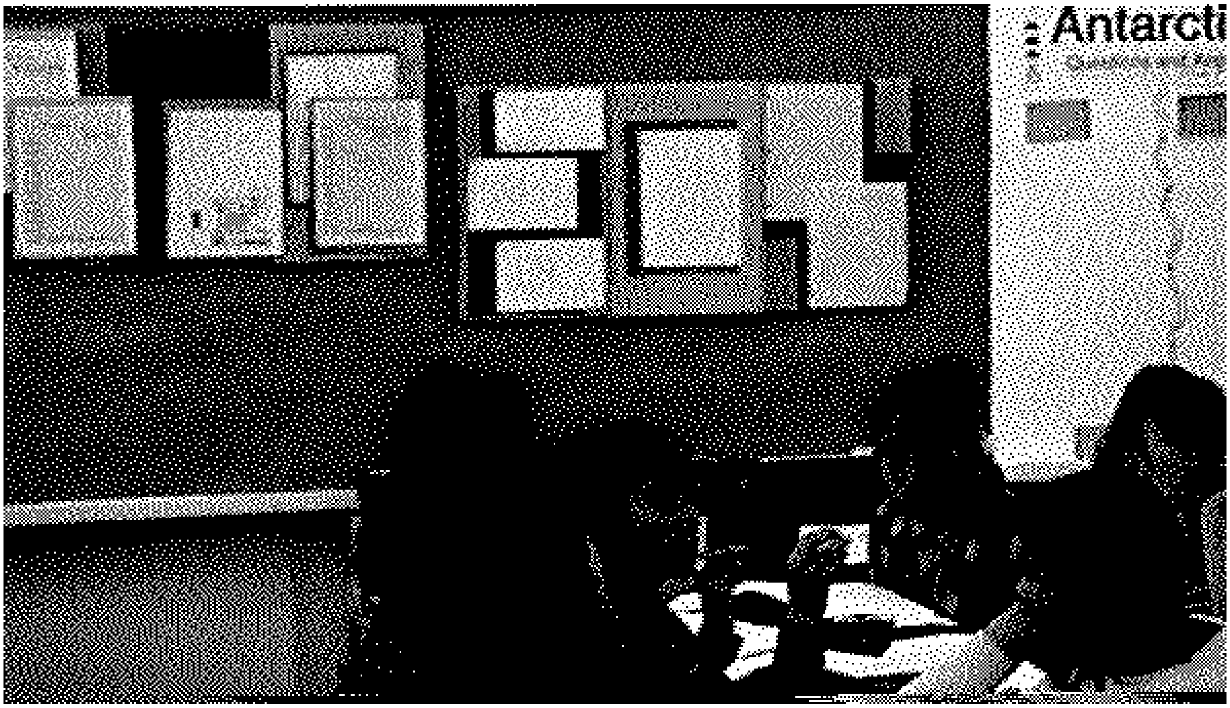
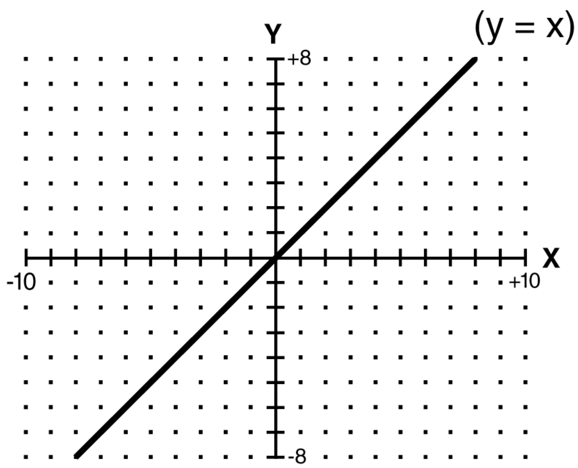


Figure 3: Problem for Example 2

8a. If you change the equation y=x to y=-0.6x,
how would the line change?



A. The steepness would change.
Why or why not?

_____NO

____YES

___ STEEPER

___ LESS STEEP

Figure 4: Lines Drawn by Marcela and Giselda

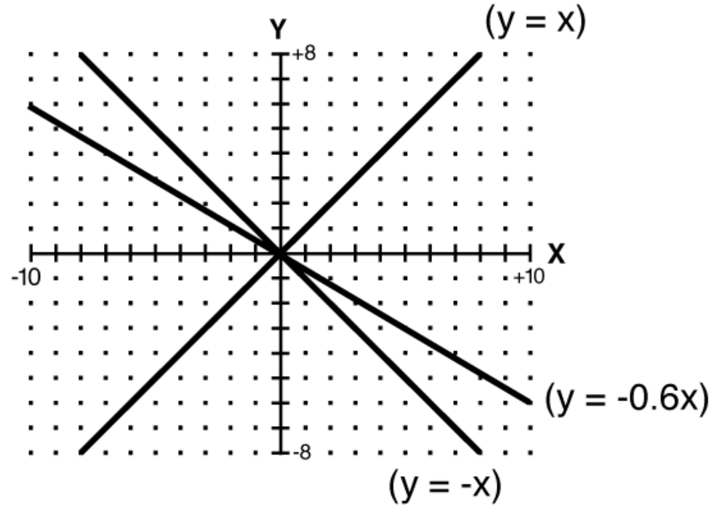


Figure 5: Comparing Assumptions of the Three Views

VOCABULARY	MULTIPLE MEANINGS	PARTICIPATION
1. Language: Mathematical communication is principally about vocabulary.	1. Language: Mathematical communication involves the mathematics register and multiple meanings for words, phrases, and constructions.	1. Language: Mathematical communication involves more than words, registers, or multiple meanings; it also involves non-language resources and discourse practices.
2. Bilingual learners are deficient in vocabulary proficiency when compared to monolinguals or native speakers.	2. Bilingual learners face additional difficulties and complications in learning to use the mathematics register and sorting out multiple meanings.	2. Bilingual learners: While bilingual learners are different than monolinguals, they are not deficient; they bring competencies and use resources. These competencies and resources may be the same or different than monolinguals.
3. Instruction should focus on developing students' vocabulary, perhaps as a pre-requisite for further study or for the study of particular mathematical topics.	3. Instruction: Since the mathematics register and multiple meanings are the main obstacles, instruction should focus on developing students' mathematics register and awareness of multiple meanings.	3. Instruction should focus on uncovering student competencies and resources and building on these.

[1] Although there are differences between the labels "bilingual" and "English Learners" for the sake of simplicity I will use "bilingual" to refer to both populations.

[2] I will use the term "sociocultural" to refer to a view of learning as inherently social, cultural, and situated. I use "situated" to mean "local, grounded in actual practices and experiences" (Gee, 1999, p. 40). Although in a previous article (Moschkovich, 2002b) I used the term "situated/sociocultural", for the sake of brevity I will use the term "sociocultural."

[3] In Moschkovich (2004) I proposed these three views as useful for understanding the relationship between learning mathematics and language. These perspectives are not meant to represent any one researcher, theorist, or school. Instead, I offer them as composite summaries of three theoretical stances reflected in work in this area. In critiquing the "acquiring vocabulary" and "constructing multiple meanings" perspectives, my purpose is not to point out how previous work was "right" or "wrong" but to examine the limitations we face when using these two perspectives.

[4] The notion of register should not be interpreted as a list of technical words and phrases. This interpretation reduces the concept of mathematical register to vocabulary and disregards the role of meaning in learning to communicate mathematically (Moschkovich, 1998).

[5] Although the word does not exist in Spanish, it might be best translated as "rangle," perhaps a shortening of the word "rectángulo."