6. HOLOGRAPHY

6.1. Gabor's (In-line) Holography.

In 1948, Dennis Gabor introduced "A new microscopic principle"[1], which he termed *holography* (from Greek *holos,* meaning "whole" or "entire", and *grafe,* "writing"). The name was chosen to indicate that the method records the entire field information (i.e. amplitude *and* phase) not just the usual intensity. Initially Gabor proposed this technique to "read" optically electron micrographs that suffered from severe spherical aberrations [1]. Nevertheless, the proof of principle demonstration was performed entirely in the optical domain and, in fact, holography has remained since largely connected with optical fields. In 1971, Gabor was awarded the Nobel Prize in Physics "for his invention and development of the holographic method".

Holography is a two-step process: 1) *writing* the hologram, which involves recording on film the amplitude and phase information, and 2) *reading* the hologram, by which the hologram is illuminated with reference field similar to that in step 1. Gabor's original setup for writing the hologram is described in Fig. 1.

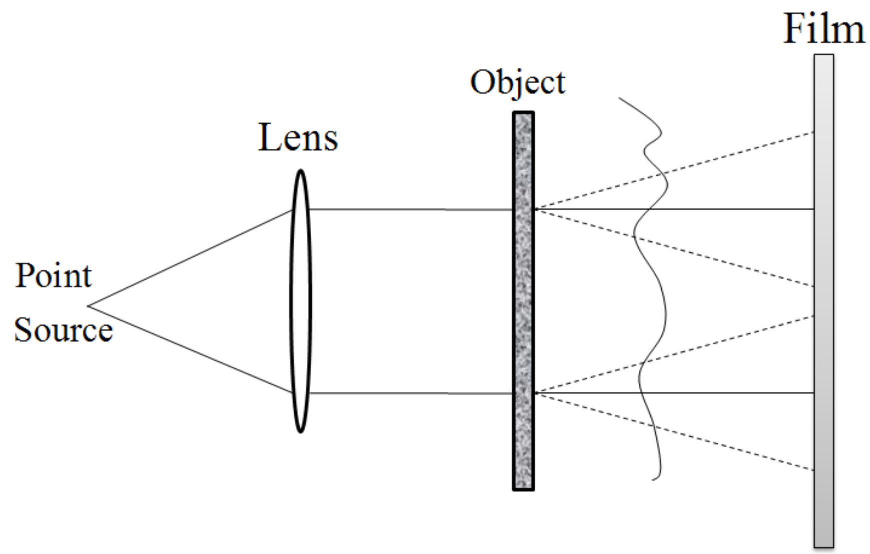

Figure 1. In-line optical setup for writing Fresnel holograms.

A point source of monochromatic light is collimated by a lens and the resulting collimated beam illuminates the semitransparent object. Note that Gabor's experiments predate the invention of lasers by more than 12 years. Thus, the light source used in these initial experiments was a mercury lamp, with appropriate spatial (angular) and temporal (color) filtering to increase the spatial and temporal coherence, respectively. The film records the Fresnel diffraction pattern of the field emerging from the object. As in phase contrast microscopy (Section 5.4), the light passing through a semitransparent object consists of the scattered (U_1) and unscattered field (U_0). At a distance *z* behind the object, the detector (photographic film during Gabor's time) records an intensity distribution generated by the interference of these two fields,

$$\begin{aligned} I(x,y) &= \left|U_0 + U_1(x,y)\right|^2 \\ &= \left|U_0\right|^2 + \left|U_1(x,y)\right|^2 + U_0 \cdot U_1^*(x,y) + U_0^* \cdot U_1(x,y). \end{aligned} \quad 1$$

Assuming a *linear response* to intensity associated with the photographic film, we find that its transmission function has the form

$$t(x,y) = a + bI(x,y), \quad 2$$

where *a* and *b* are constants. Thus, the hologram is now *written* and all the necessary information about the object is in the transmission function *t*.

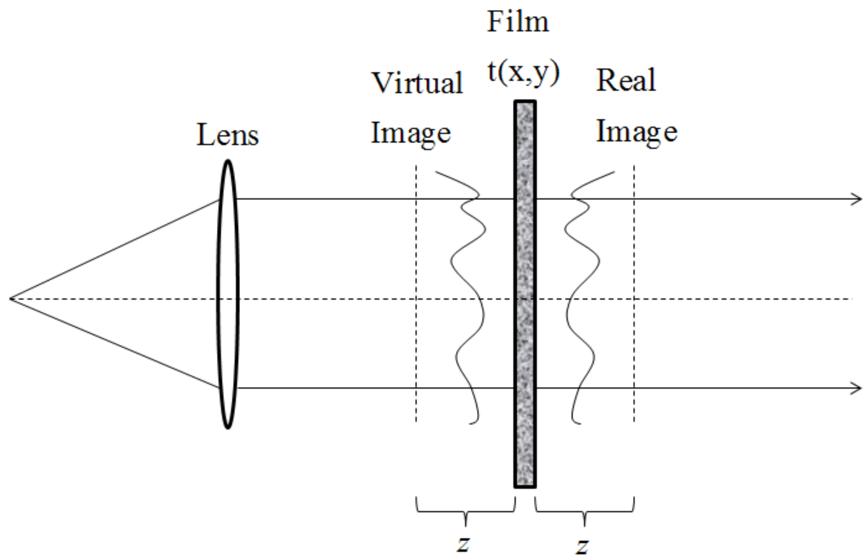

Figure 2. Reading an in-line hologram.

Reading the hologram essentially means illuminating the hologram as if it is a new object (Fig. 2). The field scattered from the hologram is the product between the illuminating plane wave (assumed to be U_0) and the transmission function,

$$\begin{aligned} U(x,y) &= U_0 \cdot t(x,y) \\ &= U_0\left(a + b\left|U_0\right|^2\right) + bU_0 \cdot \left|U_1(x,y)\right|^2 \\ &\quad + b\left|U_0\right|^2 \cdot U_1(x,y) + bU_0^{\,2} \cdot U_1^*(x,y) \end{aligned} \qquad 3$$

In Eq. 3, the first term is spatially constant, and the second term, $bU_0|U_1(x,y)|^2$ is negligible compared to the last two terms because, for a transparent object, the scattered field is much weaker than the unscattered field, $|U_0| \gg |U_1(x,y)|$. Thus, the last two terms of Eq. 3 are the relevant ones. Remarkably, these terms contain the complex field U_1 and its U_1^*. Therefore, an observer positioned behind the hologram will see at position *z* behind the transparency an image that resembles the original object (field U_1).

Field U^* indicates "backward" propagation, such that a second (*virtual)* image is formed at a distance *z* in front of the film. If the observer focuses on the plane of the first (real) image, she/he will see an overlap between the in-focus image and the out-of-focus ("twin") image due to propagation over a distance 2*z*. This overlap significantly degrades the signal to noise of the reconstruction and represents the main drawback of in-line holography. This is the reason why Gabor apparently abandoned holography by the mid 1950's [2].

In summary, in-line holography can be summarized as the process of recording the Fresnel diffraction pattern of the object onto a photosensitive film. The visualization is the reverse process by which the hologram is illuminated with a plane wave and the resulting field observed at the same Fresnel distance away. The existence of the twin images in essence is due to the hologram being a real signal, the Fourier transform of which must be an even function, i.e. symmetric with respect to the film position. In the following section, we discuss the method that circumvented the obstacle posed by the twin image formation and turned holography into a main stream technique.

6.2. Leith and Upatnieks' (Off-axis) Holography.

The advancement of holography, from Gabor's initial work to the more practical implementation using the off-axis method is well captured by Adolf W. Lohmann [3]:

"To a large extent the success of holography is associated with the invention of the off-axis reference hologram by Emmett Leith and Juris Upatnieks [4, 5]. The evolution from Gabor's inline hologram [1] to off-axis holography, however, is marked by important intermediate steps, for instance, single-sideband holography [6, 7]". Lohman's own work on holography predates Leith's, but his 1956 paper has remained less known perhaps due to its publication in German [6]. In his 1962 paper, Leith acknowledges Lohman's contributions [4]: "A discussion of various similar techniques for eliminating the twin image is given by Lohmann, Optica Acta (Paris) **3**, 97 (1956). These are likewise developed by use of a communication theory approach."

Leith and Upatnieks' pioneering paper on off-axis holography was titled "Reconstructed wavefronts and communication theory" [4], suggesting upfront the transition from describing holography as a visualization method to a way of transmitting information. In full analogy to the methods of radio communication, off-axis holography essentially adds spatial modulation (i.e. *carrier frequency*) to the optical field of interest. Interestingly, Gabor himself, like most electrical engineers at the time, was familiar with concepts of theory of communication and, in fact, published on the subject even before his 1948 holography paper [8].

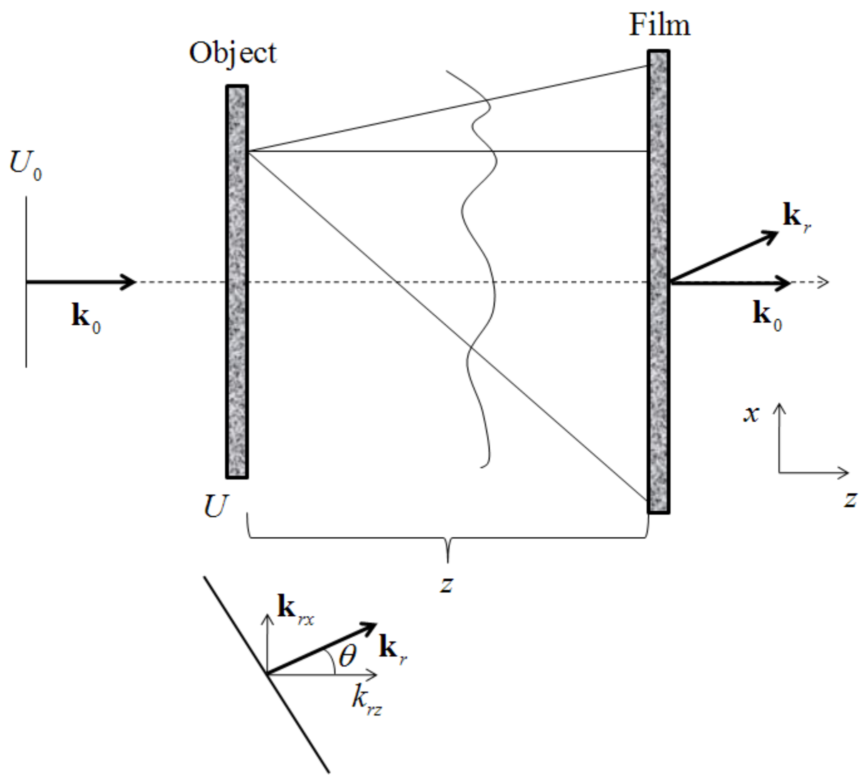

Figure 3. Off-axis setup for writing a hologram; $\mathbf{k}_0$ *and* $\mathbf{k}_r$ *are the wavevectors of the incident and reference fields.*

The principle of writing an off-axis hologram is described in Fig. 3. The object is illuminated by a monochromatic plane wave, U_0, and the transmitted field reaches the photographic film at a distance *z*. The field distribution across the film, i.e. the Fresnel diffraction pattern, $U_F(x,y)$, is a convolution between the transmission function of the object, *U*, and the Fresnel diffraction kernel (See Section 2.2. and Eqs. 2.20-21),

$$U_F(x,y) = U(x,y) * e^{\frac{ik_0\left(x^2+y^2\right)}{2z}}. \quad (4)$$

In Eq. 4, we ignored the irrelevant prefactors that do not depend on *x* and *y*.

In contrast to in-line holography, here the reference field, U_r, is delivered at an angle θ (hence *"off-axis"*) with respect to the object beam. The total field at the film plane is

$$\begin{aligned} U_t(x,y) &= U_F(x,y) + \left|U_r\right| \cdot e^{i\mathbf{k}_r\cdot\mathbf{r}} \\ &= U_F(x,y) + \left|U_r\right| \cdot e^{i\left(k_{rx}\cdot x + k_{rz}\cdot z\right)}, \end{aligned} \quad (5)$$

where $k_{rx} = k_0 \cdot \sin\theta$ and $k_{rz} = k_0 \cdot \cos\theta$.

Note that the z-component of the reference wavevector produces a constant phase shift, $k_{rz} \cdot z$, which can be ignored.

Thus the resulting transmission function associated with the hologram is proportional to the intensity, i.e.

$$\begin{aligned} t(x,y) \propto & \left|U_F(x,y)\right|^2 + \left|U_r\right|^2 \\ & + U_F(x,y) \cdot \left|U_r\right| \cdot e^{-ik_{rx} \cdot x} + U_F^{\,*}(x,y) \cdot \left|U_r\right| \cdot e^{ik_{rx} \cdot x} \end{aligned} \quad 6$$

Reading the hologram is described in Fig. 4.

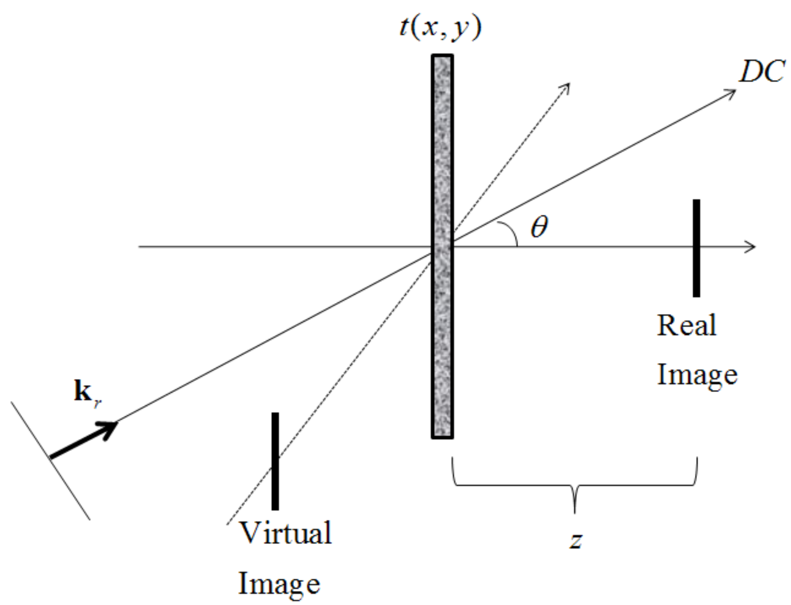

Figure 4. Reading the off-axis hologram.

Illuminating the hologram with a reference plane wave, U_r, the field at the plane of the film becomes

$$\begin{aligned} U_h(x,y) &= |U_r| \cdot e^{i\mathbf{k}_r \cdot \mathbf{r}} \cdot t(x,y) \\ &= |U_F(x,y)|^2 \cdot |U_r| \cdot e^{i\mathbf{k}_r \cdot \mathbf{r}} + |U_r|^3 \cdot e^{i\mathbf{k}_r \cdot \mathbf{r}} \\ &\quad + U_F(x,y) \cdot |U_r|^2 + U_F^*(x,y) \cdot |U_r|^2 \cdot e^{i2k_{rx} \cdot x}. \end{aligned} \quad 7$$

Equation 7 establishes that, along the optical axis, the observer has access to the complex field $U_F(x,y)$ (3rd term in Eq. 7), which, at a distance *z* from the film, reconstructs the identical replica of the object field. In other words, in reading the hologram, the free space performs the inverse operation of that in Eq. 4, that is, a *deconvolution*. This is the *real image*. The last term in Eq. 7 is modulated at a frequency $2k_{rx}$. Observing along this direction gives access to U_F^*, which reconstructs the object field behind the film, due to the complex conjugation. This is the *virtual image*.

As anticipated, the main accomplishment of this configuration is that the two images are now observed along different directions, without obstructing each other. Note that the first two terms in Eq. 7, the DC component, propagates along the direction of $\mathbf{k}_r$, which is also convenient. With the proper off-axis angle for writing/reading, the real image can be obtained *unobstructed*. In practice, the modulation frequency, k_{rx}, has to be carefully chosen to ensure the desired resolution in the final reconstruction, i.e. it must satisfy the Nyquist theorem applied to this problem. We will revisit this aspect later, when discussing off-axis methods for quantitative phase imaging (Section 8.3. and Chapter 9).

6.4. Digital Holography.

Advancement in the theory of information and computing opened the door for a new era in the field of holography. Soon after the off-axis solution to the twin image problem was proposed, it was realized that either *writing* or *reading* the hologram can be performed digitally rather than optically. In the following we discuss the basic principles of both these approaches.

6.4.1. Digital Hologram Writing.

Vander Lugt showed in 1964 that *"matched filters"* can be recorded optically and used for applications such as character recognition [14]. The idea is to calculate the cross-correlation between the known signal of interest and an unknown signal which, as a result, determine ("recognize") the presence of the first in the second.

In 1966, Brown and Lohmann of IBM showed for the first time that such a matched filter can be *written* digitally using a computer controlled plotter [15]. The goal was to calculate the autocorrelation operation in the Fourier domain, where it becomes a product (see Appendix B for this correlation theorem). Optically, the experimental setup overlaps (multiplies) the Fourier transform of the unknown signal with that of the signal of interest, as shown in Fig. 6. The unknown signal, *S*, is illuminated by a plane wave and Fourier transformed by lens L_1, where the resulting field, $\tilde{s}$, overlaps with the Fourier transform of the signal of interest, $\tilde{t}$.

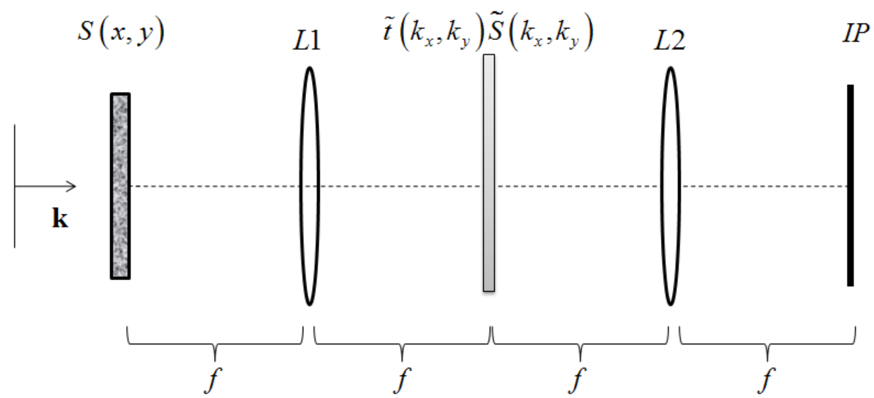

Figure 6. 4f optical system with digitally printed hologram, $t(k_x,k_y)$, in the Fourier plane; S(x,y) unknown signal, L1, L2 lenses, IP image plane.

Remarkably, Brown and Lohmann plotted the *complex* (i.e. phase and amplitude) Fourier transform of the desired signal using a binary mask, i.e. a 2D array of small dots, as used in the printing industry. The transmission function can be approximated by

$$t\left(k_x,k_y\right)=t_0+\tilde{t}_1\left(k_x,k_y\right)\cdot e^{ik_x\cdot x_0}.$$

In Eq. 9, t_0 is the DC component and $\tilde{t}_1$ is the exact Fourier transform of the signal of interest. The modulation, $e^{ik_x \cdot x_0}$, carries the spirit of off-axis holography, as it removes the twin image overlap. The field at the Fourier plane, $\tilde{U}(k_x, k_y)$, is the product between the incoming field and the mask, which acts as a diffracting object,

$$\tilde{U}(k_x, k_y) = \tilde{S}(k_x, k_y) \cdot \left[t_0 + \tilde{t}_1(k_x, k_y) \cdot e^{ik_x x_0} \right] \qquad 9$$

At the image plane (IP), the resulting field contains the cross-correlation of the fields. Thus, taking the Fourier transform of Eq. 10, we obtain

$$U(x, y) = t_0 S(x, y) + S(x, y) \otimes t_1(x - x_0, y). \qquad 10$$

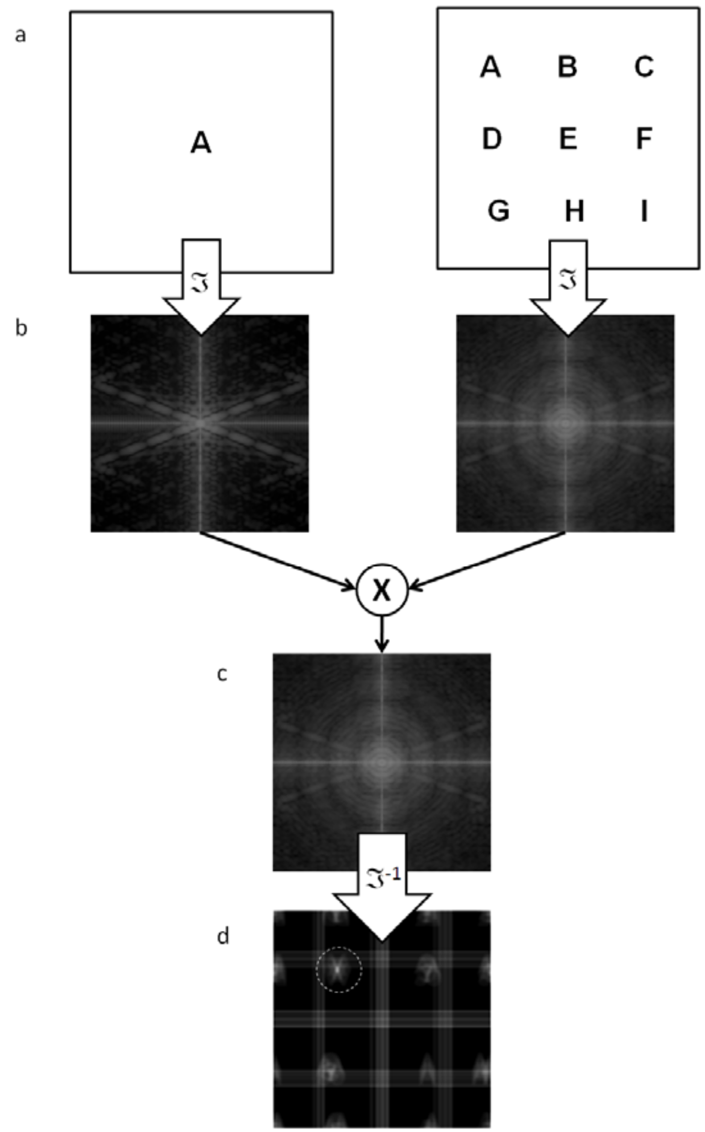

Figure 7. Character recognition using a digitally imprinted and Lugt's matched filter.

Note that due to the *shift theorem* (see Appendix B), it is straight forward to see in Eq. 10 that the modulation by $e^{ik_x \cdot x_0}$ shifts the correlation term away from the DC, which is, up to a constant, the original signal *S* (the first term on the right hand side in Eq. 11). Figure 7 illustrates this process, assuming that the signal of interest is letter A, and the unknown signal is made of 9 letters place in a 3x3 matrix. A digital mask containing the Fourier transform of the letter A is multiplied by the Fourier transform of the unknown signal. The Fourier transform of this product, i.e. the *cross-correlation*, is shown in the bottom panel. Here the highest signal corresponds to the autocorrelation function of signal A with itself. Hence, character A is recognized within the unknown signal.

6.4.2 Digital Hologram Reading.

In 1967, J. W. Goodman and R. W. Lawrence reported “Digital image formation from electronically detected holograms” [16]. A *vidicon* (camera tube) was used to record an off-axis hologram. Numerical processing was based on the fast Fourier transform algorithm proposed two years earlier by Cooley and Tukey [17]. The principle of this pioneering measurement is described in Fig. 8.

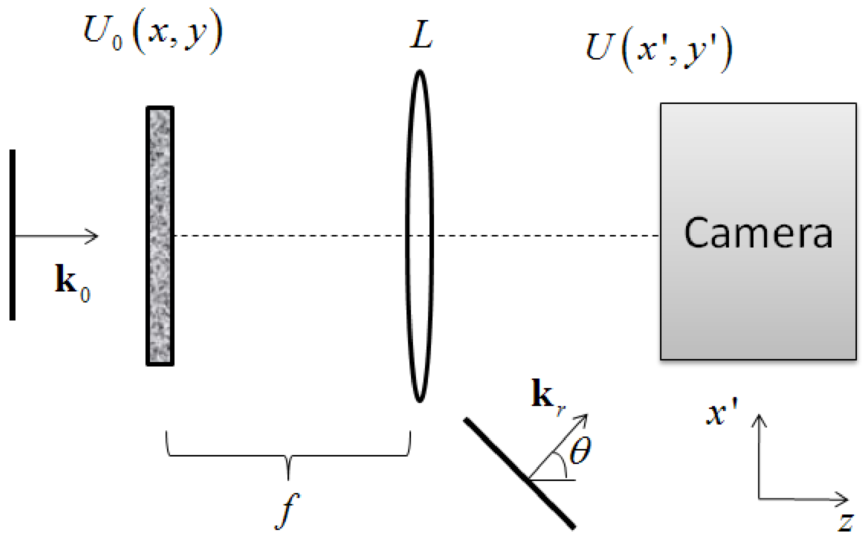

Figure 8. Digital recording of a Fourier hologram.

The transparency containing the signal of interest is illuminating by a plane wave. The emerging field, U_0, is Fourier transformed by the lens at its back focal plane, where the 2D array is positioned. The off-axis reference field U_r is incident on the detector at an angle θ. The total field at the detector is

$$\begin{aligned} U(x',y') &= \tilde{U}_0(k_x,k_y) + |U_r| \cdot e^{i\mathbf{k}_r \cdot \mathbf{r}'} \\ k_x &= 2\pi x'/\lambda f; \quad k_y = 2\pi y'/\lambda f \end{aligned}, \qquad 11$$

where $\tilde{U}_0$ denotes the Fourier transform of U_0.

The intensity that is recorded has the form

$$\begin{aligned} I(x', y') &= \left|\tilde{U}_0(k_x, k_y)\right|^2 \\ &= \left|\tilde{U}_0(k_x, k_y)\right|^2 + \left|U_r\right|^2 + \\ &+\tilde{U}_0(k_x, k_y) \cdot \left|U_r\right| \cdot e^{-ik_{rx} \cdot x'} + \tilde{U}_0^*(k_x, k_y) \cdot \left|U_r\right| \cdot e^{ik_{rx} \cdot x'}, \end{aligned} \qquad 12$$

This intensity distribution, which was recorded *digitally*, is now Fourier transformed *numerically*. Note that, due to the modulation $e^{ik_{rx} \cdot x'}$, the last two terms in Eq. 13 generate Fourier transforms that are shifted symmetrically with respect to the origin. The first two terms in Eq. 12 are not modulated and, thus, represent the DC component of the signal. Figure 9 illustrates this procedure using Goodman and Lawrence's original results [16].

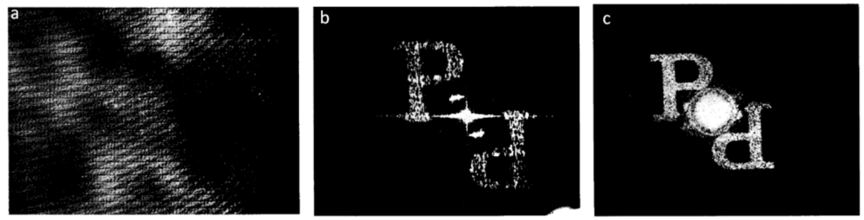

Figure 9. a) Hologram stored in computer memory; b) Display of the image reconstructed by digital computation; c) Image obtained optically from a photographic hologram, shown for comparison. Ref. [16].

Of course this numerical reconstruction is analogous to the optical one described in Section 6.4.1, where we discussed Vander Lugt's matched filter. Thus, if instead of the plane wave reference we had an arbitrary field interfering with the object field, the numerical reconstruction would yield the correlation between the Fourier transform of the two fields. However, since the reference field is a plane wave, the Fourier transform of which is a δ-function, this correlation operation returns the original signal.

In sum, in this chapter we presented a succinct review of the holography concept development, to be later used as background for the more recent developments in QPI. Since its invention, holography has become a vast field with many applications (see for example several books on the subject [18-20]). In particular, holographic microscopy has been recognized early on as a potentially powerful tool for biology (see for example a review in vol. 2, Chapter

11 of Ref. [21]). In the following chapters, we will focus mainly on the recent developments made possible by technological progress in digital recording devices. The applications of interest will in biology and medicine.

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