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Fully Fuzzy System of Linear Equations

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Abstract— In real life applications which are represented by system of linear equations, that can arise in various fields of Engineering and Sciences like Electrical, Civil, Economics, Dietary etc., there may be situation when the values of the parameters are not known or they cannot be stated precisely only their estimation due to experts knowledge is available. In such situation it may be convenient to represent such parameters by fuzzy numbers, as in [5]. Klir in [4] gave the results for the existence of solution of linear algebraic equation involving fuzzy numbers. On the similar lines we give the results for system of linear equations with fuzzy parameters. The α-cut technique is well known in obtaining weak solutions, as in [1] for fully fuzzy systems of linear equations (FFSL). However, our results state and prove the conditions for the existence and uniqueness of the fuzzy solution.

Index Terms— α-cut, Fully Fuzzy linear equations, Fuzzy numbers, level cut, weak solutions.

I. INTRODUCTION

The primitive applications in Science and Engineering give rise to system of linear equations. Many times the practical realization of such problems involves the imprecise and unknown parameters. For obtaining their solutions it is convenient to treat the coefficients and the resources, right hand side (RHS) in the equations as fuzzy numbers (e.g. Triangular, Trapezoidal, Gaussian etc.). Since FFSL with fuzzy parameters can model the applications in more realistic manner they have become more pervasive in various fields instead of their crisp counter parts.

Freidman et al., as in [3], were first to propose the model for solving the fuzzy systems with the crisp coefficient matrix and fuzzy right-hand side column. Various authors have studied these models and gave analytical as well as numerical techniques to solve such equations.

Fully fuzzy system of linear equations is important class of systems wherein all the parameters involved are fuzzy. In literature we have solution for such systems using numerical techniques, QR decomposition, LU decomposition etc. Recently in an article, [1] author have made a note that technique, as in [3] gives only the weak solutions for the fuzzy systems wherein some systems may not have the fuzzy solution.

In this paper, we consider the fully fuzzy linear system of the form $\tilde{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$, wherein the coefficients and RHS both, are represented by fuzzy numbers. We state and prove the existence and uniqueness of the fuzzy solutions for such systems.

The paper is organized in the following manner, initially the preliminaries are listed, and then the reduction of fuzzy system into the crisp using α-cut is computed. Next section gives our main results. The results are substantiated with two illustrations one satisfying the condition and hence having fuzzy solution and the otherwise. The results presented here can be generalized to n dimensional system without loss of generality.

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II. PRELIMINARIES

A. *Fuzzy number*

Let us denote by R_F the class of fuzzy subsets of the real axis (i.e. $u : R \rightarrow [0, 1]$) satisfying the following properties:

(i) $\forall u \in R_F$, u is normal, i.e. $\exists x_0 \in R$ with $u(x_0) = 1$;

(ii) $\forall u \in R_F$, u is convex fuzzy set, That is,
$u(tx + (1-t)y) \geq \min\{u(x), u(y)\}, \forall t \in [0,1], x, y \in R_F$ $\in$

(iii) $\forall u \in R_F$, u is upper semi-continuous on R;

(iv) $\overline{\{x \in R; u(x) > 0\}}$ is compact, where $\overline{A}$ denotes the closure of A.

Then R_F is called the space of fuzzy numbers, as in [2]. Obviously $R \subset R_F$ as R can be regarded as $\{\chi_x : x$ is any usual real number$\}$.

B. *Definition: Non negative fuzzy number*

A fuzzy number $\tilde{a}$ is said to be non-negative (non-positive) fuzzy number if $\mu_{\tilde{A}}(x) = 0, \forall x < 0\ (\forall x > 0)$. $\tilde{a}$ is said to be positive (negative) if $\tilde{a} \neq \tilde{0}$ and non-negative (non-positive).

C. *Definition: Fuzzy matrix*

A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. $\tilde{A}$ is positive (negative) and denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$) if each element of $\tilde{A}$ be positive (negative). $\tilde{A}$ is non-positive (non-negative) and denoted by $\tilde{A} \leq 0$ ($\tilde{A} \geq 0$) if each element of $\tilde{A}$ be non-positive (non-negative).

D. *Operations on Fuzzy Numbers:*

1. *α-cut / level-cut of fuzzy number:*

For $0 < \alpha < 1$, denote the α-cut as, $[u]^\alpha = \{x \in R : u(x) \geq \alpha\}$ and $[u]^0 = \overline{\{x \in R : u(x) > 0\}}$.

Then for each, $\alpha \in [0,1]$, the α-cut, $[u]^\alpha$ is a bounded closed interval $\left[{}^{\alpha}\underline{u}, {}^{\alpha}\overline{u}\right]$ in R.

2. *Addition and scalar multiplication:*

For $u, v \in R_F$, and $\lambda \in R$, the sum $u + v$ and the product $\lambda \cdot u$ are defined as, $[u+v]^\alpha = [u]^\alpha + [v]^\alpha$; $[\lambda \cdot u]^\alpha = \lambda [u]^\alpha$, $\forall \alpha \in [0,1]$, where $[u]^\alpha + [v]^\alpha$ means the usual addition of two intervals of R and $\lambda [u]^\alpha$ means the usual product between a scalar and a subset of R, as in [2].

System reduction using level-cuts

Consider the fully fuzzy system in 2-dimension,

$$\tilde{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}} \qquad (1)$$

i.e.

$$\begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{pmatrix} \otimes \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

Using the α-cut of the fuzzy elements, we get $\forall\alpha\in[0,1]$,

$$^{\alpha}\tilde{A}\,^{\alpha}\tilde{\mathbf{x}}={}^{\alpha}\tilde{\mathbf{b}}$$

That is,

$$\left(\begin{bmatrix}[^{\alpha}\underline{a}_{11},{}^{\alpha}\overline{a}_{11}] & [^{\alpha}\underline{a}_{12},{}^{\alpha}\overline{a}_{12}]\\ [^{\alpha}\underline{a}_{21},{}^{\alpha}\overline{a}_{21}] & [^{\alpha}\underline{a}_{22},{}^{\alpha}\overline{a}_{22}]\end{bmatrix}\right)\otimes\left(\begin{bmatrix}[^{\alpha}\underline{x}_{1},{}^{\alpha}\overline{x}_{1}]\\ [^{\alpha}\underline{x}_{2},{}^{\alpha}\overline{x}_{2}]\end{bmatrix}\right)=\left(\begin{bmatrix}[^{\alpha}\underline{b}_{1},{}^{\alpha}\overline{b}_{1}]\\ [^{\alpha}\underline{b}_{2},{}^{\alpha}\overline{b}_{2}]\end{bmatrix}\right)$$

which becomes,

$$[^{\alpha}\underline{a}_{11},{}^{\alpha}\overline{a}_{11}]\otimes[^{\alpha}\underline{x}_{1},{}^{\alpha}\overline{x}_{1}]\oplus[^{\alpha}\underline{a}_{12},{}^{\alpha}\overline{a}_{12}]\otimes[^{\alpha}\underline{x}_{2},{}^{\alpha}\overline{x}_{2}]=[^{\alpha}\underline{b}_{1},{}^{\alpha}\overline{b}_{1}]$$

$$[^{\alpha}\underline{a}_{21},{}^{\alpha}\overline{a}_{21}]\otimes[^{\alpha}\underline{x}_{1},{}^{\alpha}\overline{x}_{1}]\oplus[^{\alpha}\underline{a}_{22},{}^{\alpha}\overline{a}_{22}]\otimes[^{\alpha}\underline{x}_{2},{}^{\alpha}\overline{x}_{2}]=[^{\alpha}\underline{b}_{2},{}^{\alpha}\overline{b}_{2}]$$

Using the operations on the cut intervals the above system can be put into crisp system of linear equations as,

$$^{\alpha}\underline{a}_{11}\,{}^{\alpha}\underline{x}_{1}+{}^{\alpha}\underline{a}_{12}\,{}^{\alpha}\underline{x}_{2}={}^{\alpha}\underline{b}_{1}$$
$$^{\alpha}\underline{a}_{21}\,{}^{\alpha}\underline{x}_{1}+{}^{\alpha}\underline{a}_{22}\,{}^{\alpha}\underline{x}_{2}={}^{\alpha}\underline{b}_{2}$$
$$^{\alpha}\overline{a}_{11}\,{}^{\alpha}\overline{x}_{1}+{}^{\alpha}\overline{a}_{12}\,{}^{\alpha}\overline{x}_{2}={}^{\alpha}\overline{b}_{1}$$
$$^{\alpha}\overline{a}_{21}\,{}^{\alpha}\overline{x}_{1}+{}^{\alpha}\overline{a}_{22}\,{}^{\alpha}\overline{x}_{2}={}^{\alpha}\overline{b}_{2}$$

For each $\alpha\in[0,1]$ the system can be solved as:

$$\begin{pmatrix}^{\alpha}\underline{a}_{11} & ^{\alpha}\underline{a}_{12} & 0 & 0\\ ^{\alpha}\underline{a}_{21} & ^{\alpha}\underline{a}_{22} & 0 & 0\\ 0 & 0 & ^{\alpha}\overline{a}_{11} & ^{\alpha}\overline{a}_{12}\\ 0 & 0 & ^{\alpha}\overline{a}_{21} & ^{\alpha}\overline{a}_{22}\end{pmatrix}\begin{pmatrix}^{\alpha}\underline{x}_{1}\\ ^{\alpha}\underline{x}_{2}\\ ^{\alpha}\overline{x}_{1}\\ ^{\alpha}\overline{x}_{2}\end{pmatrix}=\begin{pmatrix}^{\alpha}\underline{b}_{1}\\ ^{\alpha}\underline{b}_{2}\\ ^{\alpha}\overline{b}_{1}\\ ^{\alpha}\overline{b}_{2}\end{pmatrix}$$

Using,

$$A=\begin{pmatrix}^{\alpha}\underline{a}_{11} & ^{\alpha}\underline{a}_{12} & 0 & 0\\ ^{\alpha}\underline{a}_{21} & ^{\alpha}\underline{a}_{22} & 0 & 0\\ 0 & 0 & ^{\alpha}\overline{a}_{11} & ^{\alpha}\overline{a}_{12}\\ 0 & 0 & ^{\alpha}\overline{a}_{21} & ^{\alpha}\overline{a}_{22}\end{pmatrix},\ \mathbf{x}=\begin{pmatrix}^{\alpha}\underline{x}_{1}\\ ^{\alpha}\underline{x}_{2}\\ ^{\alpha}\overline{x}_{1}\\ ^{\alpha}\overline{x}_{2}\end{pmatrix}\text{ and }\mathbf{b}=\begin{pmatrix}^{\alpha}\underline{b}_{1}\\ ^{\alpha}\underline{b}_{2}\\ ^{\alpha}\overline{b}_{1}\\ ^{\alpha}\overline{b}_{2}\end{pmatrix}$$

We get crisp system of the form

$$A\times\mathbf{x}=\mathbf{b} \tag{2}$$

Where, the coefficient matrix, A is of dimension $2n\times 2n$, $\mathbf{x}$ and $\mathbf{b}$ are column vectors of dimension $2n$.

We know that the unique solution of the above system exist if A invertible. Thus the necessary condition for obtaining the fuzzy solution for the fully fuzzy system is that the coefficient matrix of converted crisp system (2) is invertible $\forall\alpha\in[0,1]$.

The solution of the crisp system determines $^{\alpha}\underline{x}_{i}$ and $^{\alpha}\overline{x}_{i}$, for i = 1,2. These are used to reconstruct the components of fuzzy $\tilde{\mathbf{x}}$ as shown by the lemma below.

E. Lemma: The i^{th} component of the fuzzy solution vector, $\tilde{\mathbf{x}}$ of the fully fuzzy system (1) can be reconstructed form the components $^{\alpha}\underline{x}_{i}$ and $^{\alpha}\overline{x}_{i}$ of the crisp system (2) and is given as

$$\tilde{x}_i=\bigcup_{\alpha\in[0,1]}{}_{\alpha}\tilde{x}_i$$

Where, ${}_{\alpha}\tilde{x}_i=\alpha\cdot{}^{\alpha}\tilde{x}_i$ and ${}^{\alpha}\tilde{x}_i=\left[{}^{\alpha}\underline{x}_i,{}^{\alpha}\overline{x}_i\right]$.

Proof: For each particular $y\in R$, let $a=\tilde{x}_i(y)$.

Then

$$\left(\bigcup_{\alpha\in[0,1]}{}_{\alpha}\tilde{x}_i\right)(y)=\sup_{\alpha\in[0,1]}{}_{\alpha}\tilde{x}_i(y)$$

$$=\max\left[\sup_{\alpha\in[0,a]}{}_{\alpha}\tilde{x}_i(y),\ \sup_{\alpha\in(a,1]}{}_{\alpha}\tilde{x}_i(y)\right]$$

For each $\alpha\in(\mathrm{a},1]$, we have $\tilde{x}_i(y)=a<\alpha$ and, therefore ${}_{\alpha}\tilde{x}_i(y)=0$. On the other hand for each $\alpha\in[0,\mathrm{a}]$, we have $\tilde{x}_i(y)=a\geq\alpha$, therefore ${}_{\alpha}\tilde{x}_i(y)=\alpha$.

Hence,

$$\left(\bigcup_{\alpha\in[0,1]}{}_{\alpha}\tilde{x}_i\right)(y)=\sup_{\alpha\in[0,1]}{}_{\alpha}\tilde{x}_i(y)$$

F. Theorem (Existence and Uniqueness) The components of the solution vector x of system (2) can determine the unique fuzzy solution for system (1) if the parameters for the fuzzy system (1) satisfy the conditions:

(i) $\forall\alpha\in[0,1]$

$$\begin{pmatrix}^{\alpha}\underline{a}_{11} & ^{\alpha}\underline{a}_{12}\\ ^{\alpha}\underline{a}_{21} & ^{\alpha}\underline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\underline{b}_{1}\\ ^{\alpha}\underline{b}_{2}\end{pmatrix}\leq\begin{pmatrix}^{\alpha}\overline{a}_{11} & ^{\alpha}\overline{a}_{12}\\ ^{\alpha}\overline{a}_{21} & ^{\alpha}\overline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\overline{b}_{1}\\ ^{\alpha}\overline{b}_{2}\end{pmatrix}$$

(ii) $\forall\alpha,\beta\in[0,1],\alpha\leq\beta$

$$\begin{pmatrix}^{\alpha}\underline{a}_{11} & ^{\alpha}\underline{a}_{12}\\ ^{\alpha}\underline{a}_{21} & ^{\alpha}\underline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\underline{b}_{1}\\ ^{\alpha}\underline{b}_{2}\end{pmatrix}\leq\begin{pmatrix}^{\beta}\underline{a}_{11} & ^{\beta}\underline{a}_{12}\\ ^{\beta}\underline{a}_{21} & ^{\beta}\underline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\beta}\underline{b}_{1}\\ ^{\beta}\underline{b}_{2}\end{pmatrix}$$

$$\leq\begin{pmatrix}^{\beta}\overline{a}_{11} & ^{\beta}\overline{a}_{12}\\ ^{\beta}\overline{a}_{21} & ^{\beta}\overline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\beta}\overline{b}_{1}\\ ^{\beta}\overline{b}_{2}\end{pmatrix}\leq\begin{pmatrix}^{\alpha}\overline{a}_{11} & ^{\alpha}\overline{a}_{12}\\ ^{\alpha}\overline{a}_{21} & ^{\alpha}\overline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\overline{b}_{1}\\ ^{\alpha}\overline{b}_{2}\end{pmatrix}$$

Proof:

Existence :

Suppose condition (i) is satisfied that is $\forall\alpha\in[0,1]$

$$\begin{pmatrix}^{\alpha}\underline{a}_{11} & ^{\alpha}\underline{a}_{12}\\ ^{\alpha}\underline{a}_{21} & ^{\alpha}\underline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\underline{b}_{1}\\ ^{\alpha}\underline{b}_{2}\end{pmatrix}\leq\begin{pmatrix}^{\alpha}\overline{a}_{11} & ^{\alpha}\overline{a}_{12}\\ ^{\alpha}\overline{a}_{21} & ^{\alpha}\overline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\overline{b}_{1}\\ ^{\alpha}\overline{b}_{2}\end{pmatrix}$$

That is,

$$\left(^{\alpha}\underline{A}\right)^{-1}\cdot{}^{\alpha}\underline{\mathbf{b}}\leq\left(^{\alpha}\overline{A}\right)^{-1}\cdot{}^{\alpha}\overline{\mathbf{b}}$$

which means for i = 1,2, we have $^{\alpha}\underline{x}_i\leq{}^{\alpha}\overline{x}_i$. This implies that the interval equation $^{\alpha}\tilde{A}\,^{\alpha}\tilde{\mathbf{x}}={}^{\alpha}\tilde{\mathbf{b}}$ has a solution, which is

$$^{\alpha}\tilde{\mathbf{x}}=\left[\left(^{\alpha}\underline{A}\right)^{-1}\cdot{}^{\alpha}\underline{\mathbf{b}},\left(^{\alpha}\overline{A}\right)^{-1}\cdot{}^{\alpha}\overline{\mathbf{b}}\right].$$

Satisfaction of condition (ii) $\forall\alpha,\beta\in[0,1],\alpha\leq\beta$

$$\begin{pmatrix}^{\alpha}\underline{a}_{11} & ^{\alpha}\underline{a}_{12}\\ ^{\alpha}\underline{a}_{21} & ^{\alpha}\underline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\underline{b}_{1}\\ ^{\alpha}\underline{b}_{2}\end{pmatrix}\leq\begin{pmatrix}^{\beta}\underline{a}_{11} & ^{\beta}\underline{a}_{12}\\ ^{\beta}\underline{a}_{21} & ^{\beta}\underline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\beta}\underline{b}_{1}\\ ^{\beta}\underline{b}_{2}\end{pmatrix}$$

$$\leq\begin{pmatrix}^{\beta}\overline{a}_{11} & ^{\beta}\overline{a}_{12}\\ ^{\beta}\overline{a}_{21} & ^{\beta}\overline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\beta}\overline{b}_{1}\\ ^{\beta}\overline{b}_{2}\end{pmatrix}\leq\begin{pmatrix}^{\alpha}\overline{a}_{11} & ^{\alpha}\overline{a}_{12}\\ ^{\alpha}\overline{a}_{21} & ^{\alpha}\overline{a}_{22}\end{pmatrix}^{-1}\begin{pmatrix}^{\alpha}\overline{b}_{1}\\ ^{\alpha}\overline{b}_{2}\end{pmatrix}$$

implies that $\forall\alpha,\beta\in[0,1],{}^{\alpha}\underline{x}_i\leq{}^{\beta}\underline{x}_i\leq{}^{\beta}\overline{x}_i\leq{}^{\alpha}\overline{x}_i$ for i = 1,2.

This ensures that the solutions of the interval equations for α and β are nested; that is if $\alpha\leq\beta$, then $^{\beta}\tilde{x}_i\subseteq{}^{\alpha}\tilde{x}_i$ i.e. i^{th} component $\tilde{x}_i$ of the fuzzy number $\tilde{x}$, is convex.

Hence, $\forall\alpha\in[0,1]$, $\tilde{x}_i=\left(^{\alpha}\underline{x}_i,{}^{\alpha}\overline{x}_i\right)$, for i = 1,2. Where

- $^{\alpha}\underline{x}_i$ is a bounded left continuous non-decreasing function over [0,1].
- $^{\alpha}\overline{x}_i$ is a bounded left continuous non-increasing function over [0,1].

Thus, the solution of system (2) satisfying the above conditions would indeed generate the components of fuzzy solution vector $\tilde{\mathbf{x}}$ for the system (1).

Uniqueness :

Let $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ be two solutions of (1). Then $\forall \alpha \in [0,1]$, ${}^{\alpha}\tilde{A}{}^{\alpha}\tilde{\mathbf{x}} = {}^{\alpha}\tilde{\mathbf{b}}$ and ${}^{\alpha}\tilde{A}{}^{\alpha}\tilde{\mathbf{y}} = {}^{\alpha}\tilde{\mathbf{b}}$. Therefore, ${}^{\alpha}\tilde{A}\left({}^{\alpha}\tilde{\mathbf{x}} - {}^{\alpha}\tilde{\mathbf{y}}\right) = 0$. This implies, $\left({}^{\alpha}\tilde{\mathbf{x}} - {}^{\alpha}\tilde{\mathbf{y}}\right) = 0$. Hence $\mathbf{x} = \mathbf{y}$. Thus solution if it exists is unique. ■

Thus, the solution of the fully fuzzy linear systems having the coefficients and/or the RHS represented by fuzzy numbers exists if they satisfy the conditions given in the theorem above. The results given in the above theorem can be applied to *n* dimensional problems, without loss of generality. In the following section we give illustrations in support of our result, for ease we have considered trapezoidal fuzzy numbers. However, the results are applicable to the linear systems involving other fuzzy numbers too.

The trapezoidal fuzzy numbers *T* can be represented as (*a, b, c, d*) as shown in the Figure (1) and is mathematically represented as

$$T = \begin{cases} \dfrac{(x-a)}{(b-a)} & a < x \le b \\ 1 & b < x \le c \\ \dfrac{(d-x)}{(d-c)} & c < x \le d \\ 0 & otherwise \end{cases}$$

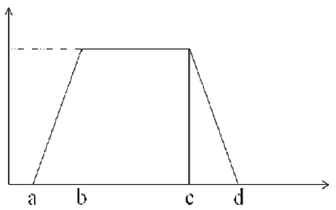

FIGURE. (1)

III. ILLUSTRATIONS

A. Example (Fuzzy Solution Exists)

Consider fully fuzzy linear system

$$(3,4,6,7)x_1 + (4,5,7,9)x_2 = (-10,20,60,97)$$
$$(3,4,6,7)x_1 + (4,5,7,9)x_2 = (-7,18,58,85)$$

Here, the coefficients and the RHS of the system are trapezoidal fuzzy numbers as given above.

The corresponding level cut system is:

$$[\alpha+3, 7-\alpha] \otimes [\underline{x}_1, \overline{x}_1] \oplus [\alpha+4, 9-2\alpha] \otimes [\underline{x}_2, \overline{x}_2] = [30\alpha-10, 97-37\alpha]$$
$$[\alpha+5, 8] \otimes [\underline{x}_1, \overline{x}_1] \oplus [3, 6-\alpha] \otimes [\underline{x}_2, \overline{x}_2] = [25\alpha-7, 85-27\alpha]$$

The corresponding crisp system obtained is

$$\begin{pmatrix} \alpha+3 & \alpha+4 & 0 & 0 \\ \alpha+5 & 3 & 0 & 0 \\ 0 & 0 & 7-\alpha & 9-2\alpha \\ 0 & 0 & 8 & 6-\alpha \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \\ {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} = \begin{pmatrix} 30\alpha-10 \\ 25\alpha-7 \\ 97-37\alpha \\ 85-27\alpha \end{pmatrix}$$

Solution of above crisp system satisfies the condition given the main theorem. Also, it gives the *α*-cuts for the components of $\tilde{\mathbf{x}}$ as:

$${}^{\alpha}\tilde{x}_1 = \left[\frac{25\alpha^2+3\alpha+2}{\alpha^2+6\alpha+11}, \frac{-\left(17\alpha^2+6\alpha+183\right)}{17\alpha^2-13\alpha-30}\right]$$

$${}^{\alpha}\tilde{x}_2 = \left[\frac{-\left(22\alpha^2+63\alpha+29\right)}{\alpha^2+6\alpha+11}, \frac{27\alpha^2+22\alpha-181}{17\alpha^2-13\alpha-30}\right]$$

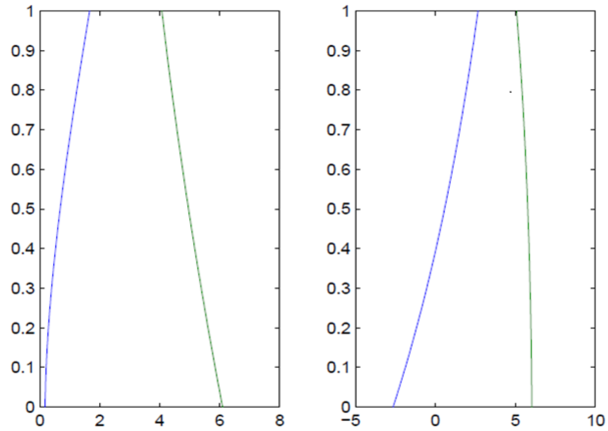

Figure. (2) Components of $\tilde{X}$

The fuzzy $\tilde{x}_1, \tilde{x}_2$ are as shown in the Figure. (2).

B. Example (Fuzzy Solution Does not Exist)

Consider another fully fuzzy linear system

$$(2,3,5,6)x_1 + (3,4,6,8)x_2 = (40,60,10,17)$$
$$(4,5,7,7)x_1 + (2,2,4,5)x_2 = (38,58,5,7)$$

The corresponding level cut system is:

$$[\alpha+2, 6-\alpha] \otimes [\underline{x}_1, \overline{x}_1] \oplus [\alpha+3, 8-2\alpha] \otimes [\underline{x}_2, \overline{x}_2] = [10\alpha+30, 77-17\alpha]$$
$$[\alpha+4, 7] \otimes [\underline{x}_1, \overline{x}_1] \oplus [2, 5-\alpha] \otimes [\underline{x}_2, \overline{x}_2] = [5\alpha+33, 65-7\alpha]$$

The corresponding crisp system obtained is:

$$\begin{pmatrix} \alpha+2 & \alpha+3 & 0 & 0 \\ \alpha+4 & 2 & 0 & 0 \\ 0 & 0 & 6-\alpha & 8-2\alpha \\ 0 & 0 & 7 & 5-\alpha \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \\ {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} = \begin{pmatrix} 10\alpha+30 \\ 5\alpha+33 \\ 77-17\alpha \\ 65-7\alpha \end{pmatrix}$$

The converted crisp system has solution. But, we can see that the conditions given in the main theorem are not satisfied as a result the fuzzy solution does not exist, it can be seen in the Figure. (3).

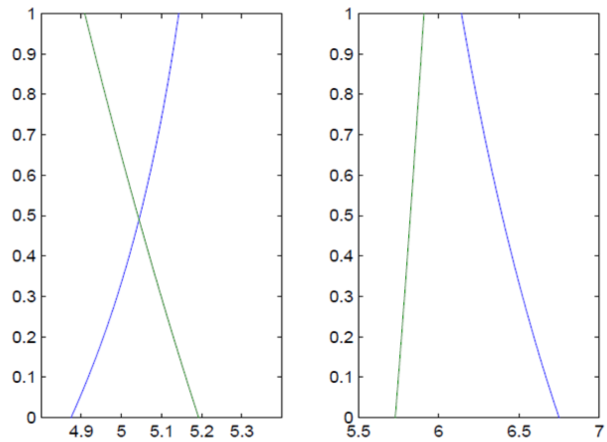

Figure. [3] Fuzzy components do not exist X

IV. CONCLUSION

In this article we extend the results for solution of fuzzy algebraic equations to the system of fully fuzzy linear equations and give the sufficient conditions for the existence and uniqueness of the solution. The illustrations are given in support of our result.

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