UCC Mathematical Tables

Calculus

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln \lvert x \rvert$	$\dfrac{1}{x}$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\cos^{-1} \dfrac{x}{a}$	$-\dfrac{1}{\sqrt{a^2-x^2}}$
$\sin^{-1} \dfrac{x}{a}$	$\dfrac{1}{\sqrt{a^2-x^2}}$
$\tan^{-1} \dfrac{x}{a}$	$\dfrac{a}{a^2+x^2}$
$\sec^{-1} \dfrac{x}{a}$	$\dfrac{a}{x\sqrt{x^2-a^2}}$
$\operatorname{cosec}^{-1} \dfrac{x}{a}$	$-\dfrac{a}{x\sqrt{x^2-a^2}}$

$f(x)$	$f'(x)$
$\cot^{-1} \dfrac{x}{a}$	$-\dfrac{a}{a^2+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$
$\coth x$	$-\operatorname{cosech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{x^2+1}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$
$\operatorname{sech}^{-1} x$	$-\dfrac{1}{x\sqrt{1-x^2}}$
$\operatorname{cosech}^{-1} x$	$-\dfrac{1}{x\sqrt{x^2+1}}$
$\coth^{-1} x$	$-\dfrac{1}{x^2-1}$

$f(x)$	$\int f(x)\,dx$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$
$\dfrac{1}{x}$	$\ln \lvert x \rvert$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$\ln \lvert \sec x \rvert$
$\sec x$	$\ln \lvert \sec x + \tan x \rvert$
$\operatorname{cosec} x$	$\ln \left\lvert \tan \dfrac{x}{2} \right\rvert$
$\cot x$	$\ln \lvert \sin x \rvert$
e^x	e^x
e^{ax}	$\dfrac{1}{a}e^{ax}$
a^x	$\dfrac{a^x}{\ln a}$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \dfrac{x}{a}$
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a} \tan^{-1} \dfrac{x}{a}$
$\dfrac{1}{x\sqrt{x^2-a^2}}$	$\dfrac{1}{a} \sec^{-1} \dfrac{x}{a}$

$f(x)$	$\int f(x)\,dx$
$\dfrac{1}{\sqrt{x^2+a^2}}$	$\ln \left\lvert \dfrac{x+\sqrt{x^2+a^2}}{a} \right\rvert$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a} \ln \left\lvert \dfrac{a+x}{a-x} \right\rvert$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\ln \left\lvert \dfrac{x+\sqrt{x^2-a^2}}{a} \right\rvert$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\coth x$	$\ln \lvert \sinh x \rvert$
$\operatorname{sech} x$	$\tan^{-1}(\sinh x)$
$\operatorname{cosech} x$	$\ln \left\lvert \tanh \dfrac{x}{2} \right\rvert$
$\cos^2 x$	$\frac{1}{2}(x + \frac{1}{2}\sin 2x)$
$\sin^2 x$	$\frac{1}{2}(x - \frac{1}{2}\sin 2x)$
$\cosh^2 x$	$\frac{1}{2}(x + \frac{1}{2}\sinh 2x)$
$\sinh^2 x$	$\frac{1}{2}(-x + \frac{1}{2}\sinh 2x)$
$\dfrac{1}{x\sqrt{a^2-x^2}}$	$-\dfrac{1}{a} \operatorname{sech}^{-1} \dfrac{x}{a}$
$\dfrac{1}{x\sqrt{x^2+a^2}}$	$-\dfrac{1}{a} \operatorname{cosech}^{-1} \dfrac{x}{a}$

Product rule $\quad y = uv \ \Rightarrow \ \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

Quotient rule $\quad y = \dfrac{u}{v} \ \Rightarrow \ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Chain rule $\quad f(x) = u\big(v(x)\big) \ \Rightarrow \ f'(x) = \dfrac{du}{dv}\dfrac{dv}{dx}$

Integration by parts $\quad \displaystyle\int u\,dv = uv - \int v\,du$

Trapezoidal rule $\quad A \approx \dfrac{h}{2}\Big[y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})\Big]$

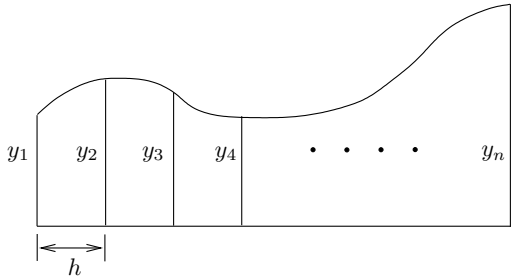
Newton-Raphson $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Taylor series (centre a) $\quad f(a+x) = f(a) + f'(a)x + \dfrac{f''(a)}{2!}x^2 + \cdots + \dfrac{f^{(r)}(a)}{r!}x^r + \cdots$

Maclaurin series $\quad f(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \cdots + \dfrac{f^{(r)}(0)}{r!}x^r + \cdots$

Volume of solid of revolution about x-axis $\quad V = \displaystyle\int_{x=a}^{x=b} \pi y^2\,dx$

Simpson's rule (n odd) $\quad A \approx \dfrac{h}{3}\Big[y_1 + y_n + 2(y_3 + y_5 + \cdots + y_{n-2}) + 4(y_2 + y_4 + \cdots + y_{n-1})\Big]$



Trigonometry

$\tan A = \dfrac{\sin A}{\cos A}$

$\cot A = \dfrac{\cos A}{\sin A} = \dfrac{1}{\tan A}$

$\sec A = \dfrac{1}{\cos A}$

$\operatorname{cosec} A = \dfrac{1}{\sin A}$

$\cos(-A) = \cos A$

$\sin(-A) = -\sin A$

$\tan(-A) = -\tan A$

$e^{in\theta} = (\cos\theta + i\sin\theta)^n$
$= \cos n\theta + i\sin n\theta$

$\cos^2 A + \sin^2 A = 1$

$\sec^2 A = 1 + \tan^2 A$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin 2A = 2\sin A\cos A$

$\cos^2 A = \dfrac{1}{2}(1 + \cos 2A)$

$\sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

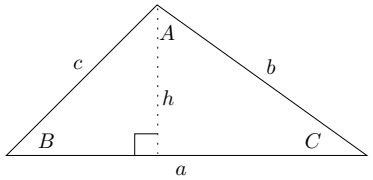
$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

Length/area/volume

Triangle

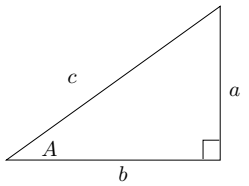


Area $A = \frac{1}{2}ab\sin C = \frac{1}{2}ah = \sqrt{s(s-a)(s-b)(s-c)}$,
where $s = \frac{1}{2}(a+b+c)$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Right-angled triangle



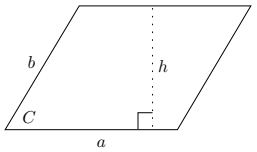
$\sin A = \dfrac{a}{c}$

$\cos A = \dfrac{b}{c}$

$\tan A = \dfrac{b}{c}$

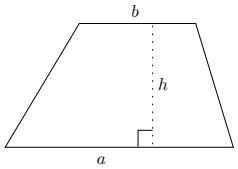
$c^2 = a^2 + b^2$

Parallelogram



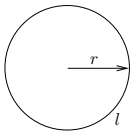
Area $A = ah = ab\sin C$

Trapezium



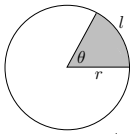
Area $A = \left(\dfrac{a+b}{2}\right)h$

Circle



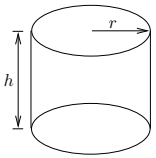
Circumference $l = 2\pi r$
Area $A = \pi r^2$

Arc/sector



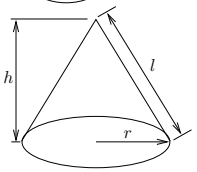
Length $l = r\theta$ (θ in radians)
Area $A = \frac{1}{2}r^2\theta$

Cylinder



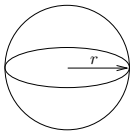
Curved surface area $A = 2\pi rh$
Volume $V = \pi r^2 h$

Cone



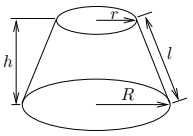
Curved surface area $A = \pi rl$
Volume $V = \frac{1}{3}\pi r^2 h$

Sphere



Surface area $A = 4\pi r^2$
Volume $V = \frac{4}{3}\pi r^3$

Frustrum of cone



Curved surface area $A = \pi(r+R)l$
Volume $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$