The Simplex Algorithm of Dantzig

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Abstract: ***In this chapter, we put the theory developed in the last to practice. We develop the simplex method algorithm for LP problems given in feasible canonical form and standard form.***

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I. INTRODUCTION

The fundamental theorem of linear programming which states that if the given linear programming problem has an optimal solution, then at least one basic feasible solution must be optimal forms a firm base for the solution of L.P. problem. According to this theorem we can search the optimal solution among the basic feasible solutions only which are finite in number. Also it is easy to find an optimal among the basic feasibles than to find that among all the feasible solutions which may be infinite in number. In this way a L.P. problem can be solved by enumerating all the B.F. solutions. But it is not an easy job to enumerate all the B.F. solutions even for small values of m (number of constraints) and n (no. of variables). To overcome this difficulty a method known as **Simplex Method** (or **Simplex Algorithm**) was developed by **George Dantzig** in 1947 which was made available in 1951. This method is an iterative procedure in which we proceed in systematic steps from an initial B.F. solution to other B.F. solutions and finally, in a finite number of steps, to an optimal B.F. solution, in such a way that the value of the objective function at each iteration is better (or at least no worse) than at the preceding step.

II. SIMPLEX METHOD FOR PROBLEMS IN FEASIBLE CANONICAL FORM

The Simplex method is a method that proceeds from one BFS or extreme point of the feasible region of an LP problem expressed in tableau form to another BFS, in such a way as to continually increase (or decrease) the value of the objective function until optimality is reached. The simplex method moves from one extreme point to one of its neighbouring extreme point. Consider the following LP in feasible canonical form, i.e. its right hand side vector $b \geq 0$:

$$\text{max} \qquad X_0 = C^T X$$

$$\text{subject to} \quad \begin{cases} A_X \leq b \\ X \geq 0 \end{cases}$$

Its initial tableau is

	x_1	x_2	$\dots$	x_S	$\dots$	x_n	x_{n-1}	$\dots$	x_{n+r}	$\dots$	x_{n+m}	**b**
x_{n-1}	a_{11}	a_{12}	$\dots$	a_{1S}	$\dots$	a_{1n}	1	$\dots$	0	$\dots$	0	b_1
x_{n-2}	a_{21}	a_{22}	$\dots$	a_{2S}	$\dots$	a_{2n}	0	$\dots$	0	$\dots$	0	b_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_{n-r}	a_{ri}	a_{r2}	$\dots$	a_{rS}	$\dots$	a_{rn}	0	$\dots$	1	$\dots$	0	b_r
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_{n+m}	a_{m1}	a_{m2}	$\dots$	a_{m3}	$\dots$	a_{mn}	0	$\dots$	0	$\dots$	1	b_m
x_0	$-c_1$	$-c_2$	$\dots$	$-c_S$	...	$-c_n$	0	$\dots$	0	$\dots$	0	0

Here X_{n+i}, i = 1 …, *m* are the slack variables. The original variables $X_i, i=1,\dots,$ n are called the structural or decision variables. Since all $b_i \geq 0$, we can read off directly from the tableau a starting BFS given by $[0, 0, \dots 0, b_1, b_2, \dots, b_m]^T$, i.e., all structural variables

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X_j are set to zero. Note that this corresponds to the origin of the n-dimensional subspace R^n of R^{n+m}.

In Matrix form, the original constraint $Ax \le b$ has be augmented to

$$[A\ I]\begin{bmatrix} x \\ x_s \end{bmatrix} = Ax + Ix_s = b \qquad (1)$$

Here x_s is the vector of slack variables. Since the columns of the augmented matrix $[A : I]$ that correspond to the slack variables $\{x_{n+i}\}^m{}_{i=1}$ is an identity matrix which is clearly invertible, the slack variables $\{x_{n+i}\}^m{}_{i=1}$ are basic. We denote by *B* the set of current basic variables, i.e. $B = \{x_{n+i}\}^m{}_{i=1}$. The set of non-basic variables i.e. $\{x_i\}^n{}_{i=1}$ will be denoted by *N*.

We consider now the process of replacing an $x_r \in B$ by an $x_8 \in N$. We say that x_r is to leave the basis and x_8 is to enter the basis. Consequently after this operation, x_r becomes non-basic, i.e.. $x_r \in N$ and x_s becomes basic, i.e. $x_s \in B$. This of course amounts to a different (selection of columns of matrix A to give a different) basis B. We shall achieve this change of basis by a pivot operation (or simply called pivot). This pivot operation is designed to maintain an identity matrix as the basis in the tableau at all time.

III. PIVOT OPERATION WITH RESPECT TO THE ELEMENT a_{rs}

Once we have decided to replace $x_r \in B$ by $x_s\ 2 \in N$, the a_{rs} in the tableau will be called the pivot element. We will see later that the feasibility condition implies that $a_{rs} > 0$. The r-th row and the s-th column of the tableau are called the pivot row and the pivot column respectively. The rules to update the tableau are:

(1) In pivot row, $a_{rj} \leftarrow a_{rj} / a_{rs}$ for $j = 1, \dots n+m$.

(2) In pivot column,

$a_{rj} \leftarrow 1, a_{is} \leftarrow 0$ for $i = 0, \dots m, i \ne r$.

(3) For all other elements, $a_{ij} \leftarrow a_{ij} - a_{rj} * a_{is} / a_{rs}$.

Graphically, we have

	j	s
i	a_{ij}	a_{is}
r	r_{rj}	a^*_{rs}

becomes

j	s
$a_{ij} - a_{ij}a_{is} / a_{rs}$	0
a_{rj} / a_{rs}	1

Or, simply,

a	b
c	d^*

becomes

$a - \frac{bc}{d}$	0
$\frac{c}{d}$	1

This pivot operation is simply the Gaussian elimination such that variable x_s is eliminated from all $m+1$ but the r-th equation, and in the r-th equation, the co-efficient of x_s is equal to 1. In fact, Rule (1) above amounts to normalization of the pivot rows such that the pivot element becomes 1. Rule (2) above amounts to eliminations of all the entries in the pivot column except the pivot element. Rule (3) is to compute the Schur's complement for the remaining entries in the tableau.

IV. SIMPLEX METHOD FOR PROBLEMS IN FEASIBLE CANONICAL FORM

Example 1.1

$$\text{Consider} \begin{cases} x_1 - x_2 - x_3 - x_4 & = 3 \\ 2x_1 - 3x_2 - x_3 \quad - x_5 & = 3 \\ -x_1 - 2x_2 - x_3 \quad + x_6 & = 1 \end{cases}$$

The Initial Tableau is given by Tableau 1:

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	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1^*	1	−1	1	0	0	5
x_5	2	−3	1	0	1	0	3
x_6	−1	2	−1	0	0	1	1

$$B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The current basic solution is $[0, 0, 0, 5, 3, 1]^T$ which is clearly feasible. Suppose we choose $a_{1,1}$ as our pivot element. Then after one pivot operation, we have

Tableau 2:

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	1	−1	1	0	0	5
x_5	0	-5^*	3	−2	1	0	−7
x_6	0	3	−2	1	0	1	6

$$B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

We note that the current basic solution is $[5, 0, 0, 0, -7, 6]^T$ which is infeasible. Using the new (2, 2) entry as pivot, we have

Tableau 3:

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	0	$-\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{18}{5}$
x_2	0	2	$-\frac{3}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{7}{5}$
x_6	0	0	$-\frac{1^*}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	1	$\frac{9}{5}$

$$B_3 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -3 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

The current basic solution is $\left[\frac{18}{5}, \frac{7}{5}, 0, 0, 0, \frac{9}{5}\right]^T$ and is feasible. Finally, let us eliminate the last slack variable x_6 by replacing it by x_3.

Tableau 4:

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	0	0	1	−1	−2	0
x_2	0	1	0	1	−2	−3	−4
x_3	0	0	1	1	−2	−5	−9

$$B_1 = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \\ -1 & 2 & -1 \end{bmatrix}$$

The current basic solution is $[0, -4, -9, 0, 0, 0]^T$ which is infeasible and degenerate. Thus we see that one cannot choose the pivot arbitrarily. It has to be chosen according to some feasibility criterion. There are three important observations that we should not here. First the pivot operations which amounts to elementary row operations on the tableaus, are being recorded in the tableaus at the columns that correspond to the slack variables. In the example above, one can easily check that Tableau I is obtained from Tableau 1 by pre-multiplying Tableau 1 by the matrix formed by the columns of x_4, x_5 and x_6 in Tableau *i*. In the Tableaus, the inverse of

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these matrices are computed and are denoted by B_i. Let the columns in Tableau 1 be denoted as usual by a_j and the columns in Tableau i be denoted by y_j, then since

$$[a_1, a_2, \ldots, a_{m+n}] = [A \vdots I] = B_i [B_i^{-1} A \vdots B_i^{-1}] = B_i [Y_1, Y_2, \ldots, Y_{m+n}]$$

It is clear that $B_i Y_j = a_j$. Comparing this with equation (2.20), we see that B_i are the change of basis matrices from Tableau 1 to Tableau 4.

Our second observation is the following one. Since the last column in Tableau 1 is given by b, the last column in Tableau *I*, which we denote by $y_0 = (y_{10}, \ldots y_{m0})^T$, will be given by $B_i y_0 = b$. Since B_i is invertible, y_0 gives the basic variables of the current basic solution, i.e. the basic solution x^i B corresponding to B_i is given by

$$x_B^i = y_0 = B_i^{-1} b$$

For this reason, the last column of the tableau, i.e. y_0, is called the solution column.

The third observation is that the columns of B_i are the columns of the initial tableau. For example, the columns of B_3 are the first, second and the sixth columns of Tableau 1. In fact, Tableau 3 is obtained by moving (via elementary row operations) the identity matrix in Tableau 1 to the first, second and the sixth columns in Tableau 3. It indicates that in each iteration of the simplex method, we are just choosing different selection of columns of the augmented matrix to give a different basic matrix B. In particular, the solution obtained in each tableau is indeed the basic solution to our original augmented matrix system (3, 1). In fact, Tableau 3 means that

$$[A \vdots I] = \begin{bmatrix} \frac{18}{5} \\ \frac{7}{5} \\ 0 \\ 0 \\ 0 \\ \frac{9}{5} \end{bmatrix} = B_3 \begin{bmatrix} \frac{18}{5} \\ \frac{7}{5} \\ \frac{9}{5} \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = b$$

i.e. the current solution is given by $\left[\frac{18}{5}, \frac{7}{5}, 0, 0, 0, \frac{9}{5}\right]^T$.

For Tableau 4, since $B_4 = A$, we have

$$[I \vdots A^{-1}] \begin{bmatrix} A^{-1}b \\ 0 \end{bmatrix} = A^{-1}b$$

Which is equivalent to

$$A[A^{-1}]_0 - I \cdot 0 = b$$

i.e. the current solution in Tableau 4 is given by $[A^{-1}b, 0]^T$

In the following, we consider the criteria that guarantee the feasibility and optimality of the solutions.

V. FEASIBILITY CONDITION

Suppose that the entering variable x_s has been chosen according to some optimality conditions, i.e. the pivot column is the s-the column. Then the leaving basic variable x_r must be selected as the basic variable corresponding to the smallest positive ratio of the values of the current right hand side to the current positive constraint co-efficient of the entering non-basic variable x_s.

To determine row r	x_s	y	$Ratio$
	y_{1s}	y_{10}	$\frac{y_{1s}}{y_{10}}$
	y_{2s}	y_{20}	$\frac{y_{2s}}{y_{20}}$
	$\vdots$	$\vdots$	$\vdots$
$\frac{y_{rb}}{y_{rx}} = \min_i \left\{ \frac{y_{rb}}{y_{rx}}, y_{rs} > 0 \right\}$	y_{is}	y_{i0}	$\frac{y_{is}}{y_{i0}}$
	$\vdots$	$\vdots$	$\vdots$
	y_{ms}	y_{m0}	$\frac{y_{ms}}{y_{m0}}$

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VI. OPTIMALITY CONDITION

For simplicity, we consider a maximization problem. We first denote the entries in the row that correspond to x_0 by y_{0j}. The $(m+1, m+n+1)-$th entry in the tableau is denoted by y_{00}. We will show in the next section that

$$y_{ij} = -(c_j - z_j), \quad j = 1.\cdots.n-m \qquad (3)$$

The negation of the reduced cost co-efficient that appeared in Theorem 2.6. Here z_j is defined in (2.25). Moreover, we will show also that

$$y_{00} = c_B^T x_B \qquad (4)$$

i.e. y_{00} is the current objective function value associated with the current BFS in the tableau. Thus according to Theorem 2.6, the entering variable x_s 2 N can be selected as a non-basic variable x_s having a negative co-efficient. Usual choices are the first negative $y0_s$ or the most negative $y0_s$. If all co-efficient $y0_j$ are non-negative, then by Theorem 2.7, an optimal solution has been reached.

Summary of Computation Procedure

Once the initial tableau has been constructed, the simplex procedure calls for the successive iteration of the following steps.

1. Testing of the co-efficient of the objective function row to determine whether an optimal solution has been reached, i.e., whether the optimality condition that all co-efficients are non-negative in that row is satisfied.
2. If not, select a currently non-basic variable x_s to enter the basis. For example, the first negative co-efficient or the most negative one.
3. Then determine the currently basic variable x_r to leave the basis using the feasibility condition, i.e. select x_r where $y_{r0}/y_{rs} = \min\{y_{i0}/yis\ y_{is} > 0\}$
4. Perform a pivot operation with pivot row corresponding to x_r and pivot column corresponding to x_s. Return to 1.

Example 3.2. Consider the LP problem:

$$\text{Max} \quad x_0 = 3x_1 - x_2 + 3x_3$$

$$\text{Subject to} \quad \begin{cases} 2x_1 + x_2 - x_3 \le 2 \\ x_1 + 2x_2 - 3x_3 \le 5 \\ 2x_1 + 2x_2 - x_3 \le 6 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

By adding slack variables x_4, x_5 and x_6, we have the following initial tableau.

Tableau 1: Initial tableau, current BFS is $x = [0, 0, 0, 2, 5, 6]^T$ and $x_0 = 0$.

	x_1	x_2	x_3	x_4	x_5	x_6	b	Ratio
x_4	2	1^*	2	2	0	0	2	$\frac{2}{1}=2$
x_5	1	2	3	0	1	0	5	$\frac{5}{2}=2.5$
x_6	2	2	2	0	0	2	5	$\frac{6}{2}=3$
x_0	−3	−2	−3	0	0	0	0	

We choose x_2 as the entering variable to illustrate that any non-basic variable with negative co-efficient can be chosen as entering variable. The smallest ratio is given by x_4 row. Thus x_4 is the leaving variable.

Tableau 2: Current BFS is

$x = [0, 2, 0, 0, 1, 2]^T$ and $x_0 = 2$

	x_1	x_2	x_3	x_4	x_5	x_6	b	Ratio
x_2	2	1	1	1	0	0	2	$\frac{2}{1}=2$
x_5	−3	0	1^*	−2	1	0	1	$\frac{1}{1}=1^*$
x_6	−2	0	−1	−2	0	1	2	
x_0	−1	0	−2	1	0	0	2	

Tableau 3: Current BFS is

$x = [0, 1, 1, 0, 0, 3]^T$ and $x_0 = 4$.

	x_1	x_2	x_3	x_4	x_5	x_6	b	Ratio
x_2	5^*	1	0	3	−1	0	1	$\frac{1}{5}$
x_3	−3	0	1	−2	1	0	1	
x_6	−5	0	0	−4	1	1	3	
x_0	−7	0	0	−3	2	0	4	

Tableau 4: Optimal tableau, optimal BFS

$$x^* = \left[\frac{1}{5}, 0, \frac{8}{5}, 0, 0, 4\right]^T, x^*$$

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	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	1	$\frac{1}{2}$	0	$\frac{3}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
x_3	0	$\frac{3}{2}$	1	$-\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{3}{2}$
x_6	0	1	0	-1	0	1	$\frac{1}{3}$
x_0	0	$\frac{3}{2}$	0	$\frac{5}{3}$	$\frac{3}{2}$	0	$\frac{3}{4}$

We note that the extreme point sequence that the simplex method passes through are

$$\{x_4, x_5, x_6\} \to \{x_2, x_5, x_6\} \to \{x_2, x_3, x_6\} \to \{x_1, x_3, x_6\}.$$

VII. SIMPLEX METHODS FOR PROBLEMS IN STANDARD FORM

Our previous method is based upon the existence of an initial BFS to the problem. It is desirable to have an identity matrix as the initial basic matrix. For LP in feasible canonical form, the initial basic matrix is the matrix associated with the slack variables, and is an identity matrix. Consider an LP in standard form:

$$\max \quad x_0 = c^T x$$

$$\text{subject to} \quad \begin{cases} Ax = b \\ x \geq 0 \end{cases}$$

Where we assume that $b \geq 0$. There is no obvious initial starting basis *B* such that $B = I_m$ .For notational simplicity, assume that we pick B as the last m (linearly independent) columns of *A*, i.e.

A is of the form $A = [N : B]$. We then have for the augmented system:

$$\begin{cases} N_{xy} + B_{xy} = b \\ x_a - c_y^T xy - c_y^T xy = 0 \end{cases}$$

Multiplying by B^{-1} to the first equation yields,

$$B^{-1} N_{xN} + x_B = B^{-1} b$$

$$x_B = B^{-1} b - B^{-1} Nxy$$

Hence the x_0 equation becomes

$$x_0 - c_N^T xy - c_B^T (B^{-1} b - B^{-1} NxN) = 0$$

Thus we have

$$\begin{cases} B^{-1} N x_N + x_B = B^{-1} b \\ x_0 - (c_N^T - c_B^T B^{-1} N) x_N = c_B^T B^{-1} b \end{cases}$$

Denoting $Z^T{}_N = Z^T{}_B B^{-1} N$ (an (n - m) row vector) gives

$$\begin{cases} B^{1} N x_n + x_B = B^{-1} b \\ x_0 - (c_N^T - z_N^T) x_N = c_B^T B^{-1} b \end{cases}$$

Which is called the general representation of an LP in standard form with respect to the basis B. Its initial simplex tableau is then

	x_N	x_B	b
x_B	$B^{-1} N$	I	$B^{-1} b$
x_0	$-(c_N^T - z_N^T)$	0	$c_B^T B^{-1} b$

We note that the j-th entry of Z_N is given by

$$c_B^T B^{-1} N_j = c_B^T B^{-1} a_j = c_B^T y_j = z_j$$

Where z_j is defined as in (2.21). Thus in the table, we see that the entries in the x_0 row are given by $-(c_j - z_j)\, x_j N$ and zero for $x_j \in B$. Thus they are the negation of the reduced cost co-efficients. This varies equation (3.3) that we have assumed earlier. Moreover, by (3.2), we see that

$$y_{00} = c_B^T B^{-1} b = c_B^T x_B$$

Which is the same as (4.4)

We remark that x_0 is now expressed in terms of the non-basic variables,

$$x_0 = c_B^T B^{-1} b + \sum_{x_j \in N}^{n} (c_j - z_j)\, x_j \tag{5}$$

Hence it is easy to see that for maximization problem, the current BFS is optimal when $c_j - z_j \geq 0$ for all *j*. For minimization problem, the current BFS will be optimal when $c_j - z_j$, 0 for all *j*.

Example 3.3. Consider the following LP

$$\max \quad x_0 = x_1 + x_2$$

$$\text{subject to} \quad \begin{cases} 2x_1 + x_2 \geq 4 \\ x_1 + 2x_2 = 6 \\ x_1, x_2 \geq 0 \end{cases}$$

Putting into standard form by adding the surplus variable

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x_3 the augmented system is:

$$\begin{cases} 2x_1 + x_2 - x_3 = 4 \\ x_1 + 2x_2 \quad\quad = 6 \\ x_0 - x_1 - x_2 \quad = 0 \end{cases}$$

The simplex tableau for the problem is:

Tableau 1:

	x_1	x_2	x_3	b
	2	1	−1	4
	1	2	0	6
x_0	−1	−1	0	0

Here we do not have a starting identity matrix. Suppose we let x_1 and x_2 to be our starting basic variables, then

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \ N = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \text{and } c_B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

In this case

$$B^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$x_3 = B^{-1}N = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$b = B^{-1}b = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{8}{3} \end{bmatrix}$$

It is also easily check that

$$z_3 = c^T B^{-1} a_j = [-1 \quad -1]\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \frac{1}{3}$$

And the current value of the objective function is given by

$$c_B^T B^{-1} b = [-1 \quad -1]\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \frac{10}{3}$$

Hence the starting tableau is:

Tableau 2:

	x_1	x_2	x_3	b
x_1	1	0	$-\frac{2}{3}$	$\frac{2}{3}$
x_2	0	1	$\frac{1}{3}^*$	$\frac{8}{3}$
x_0	0	0	$-\frac{1}{3}$	$\frac{10}{3}$

Thus $x = \left[\frac{2}{3}, \frac{8}{3}, 0\right]^T$ is an initial BFS. We can now apply the simplex method as discussed in x_1 to find the optimal solution. The next iteration gives:

	x_1	x_2	x_3	b
x_1	1	2	0	6
x_3	0	3	1	8
x_0	0	1	0	6

Thus the optimal solution is

We note that if we choose x_1 and x_3 as our starting basis variables, then we get Tableau 3 immediately and no iteration is required. However, if x_2 and x_3 are chosen as starting variables, then we have

Tableau 3:

	x_1	x_2	x_3	b
x_1	$\frac{1}{2}$	1	0	3
x_3	$-\frac{3}{2}$	0	1	−1
x_0	$-\frac{1}{2}$	0	0	3

Hence the starting basis solution is not feasible and we cannot use the simplex method to find our optimal solution.

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