Multiuser MIMO Downlink Scenario Based on Block Diagonalization Transmission

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Abstract—**In this paper a multiuser Multiple Input Multiple Output (MIMO) downlink transmission scheme is proposed. We utilize properties of Block Diagonalization (BD) and Successive Optimization (SO) methods for downlink transmission, and suggest a new scheme in order to enhance the achievable sum rate of the system, while different transmission rates are assigned to different service groups by using only spatial multiplexing. Simulation results show that proposed scheme increase the achievable sum rate of the system in common SNRs while the predetermined proportional rate requirements of users are obtained.**

I. INTRODUCTION

In the last decade, the information theoretic approaches for Multiple Input Multiple Output (MIMO) Multiuser communication have proven that the sum capacity of MIMO Broadcast channel (MIMO-BC) or Downlink transmission can be achieved by Dirty Paper Coding (DPC) [1]-[3]. The idea is to cancel out Interference among users at base station by precoding, assuming complete Channel State Information (CSI) at Base station. Sum capacity maximization for MIMO-BC is a convex problem and thus can be solved with iterative water-filling algorithm [4]. In practice, available systems do not have such a computational power for these enormous iterative matrix calculations. However to cope with the problem, some methods have been proposed to achieve DPC performance approximately.

More practical transmission techniques are multiuser linear pre-coding methods, in which users are multiplexed in spatial domain. The most famous linear multiuser transmission scheme is Zero Forcing (ZF) or channel inversion [6]. Zero Forcing completely eliminates the interference among transmit streams across different antennas. The precoding vectors in ZF transmitter are obtained by inversion of aggregate users channel matrix; and the maximum sum rate in this case can be obtained by using water-filling power allocation scheme over effective channels of different users [6]. The major drawback of ZF method is noise enhancement, which leads to reduction of system sum capacity.

The regularized version of ZF is Minimum Mean Square Error (MMSE) precoding which is optimal in the sense of compromising between interference cancellation and noise effect reduction [7]. MMSE outperform Zero Forcing sum capacity in all Signal to Noise Ratios (SNRs), particularly in low SNRs. However, Channel inversion schemes are not suitable for systems in which users have more than one antenna in Mobile Station (MS) because these methods cannot employ the correlation between antennas of each user, which could be used to enhance the sum capacity.

In multiuser MIMO system with single antenna per user, the goal of the system is to eliminate interference across different antennas, which each of them represent one user. On the other hand, in systems with multiple antennas at each mobile station, we want to eliminate interference only across different users and not different antennas, so that by using the cooperation between antennas of each user, it would be possible to increase the sum capacity in the system. The most well-known scheme in this class of linear pre-coding methods is Block Diagonalization (BD) [8]-[10]. In BD, pre-coding matrix of each user is designed such that the transmit signal of the user lies in the null space of all other users. Consequently, with perfect CSI at base station, the interference between users pre-removed. In order to obtain the maximum achievable rate of BD, water-filling power allocation [11] should be performed on singular values of effective channel matrices. Another well known linear method for systems with multi antennas per each mobile station is Successive Optimization (SO) [12], where pre-coding matrix of each user is designed such that it does not interfere with the users that previously processed in the system. By performing this method, we can prioritize users based on their required rates. Transmission power of each user in SO method can be individually allocated so that required rate for each user is achieved [13].

In practice, usually there are various services in telecommunication systems, and consequently there are different rate requirements based on different levels of service quality. Hence, in practical systems, it is necessary to assign different rates to different users in order to efficiently use available resources. In the methods that we mentioned above, Block Diagonalization completely eliminates Interference between users without considering any priority among users in the case of using only spatial resources. On the other hand Successive Optimization enables the system to set different transmission rates to different services, only by means of spatial multiplexing. However in Successive Optimization method, if total power is shared equally among users with the same channel gain, users that is precoded at the end, only will get small portion of overall capacity, because interference from previously processed users is not eliminated for subsequent users.

By considering the properties of these two schemes, in this

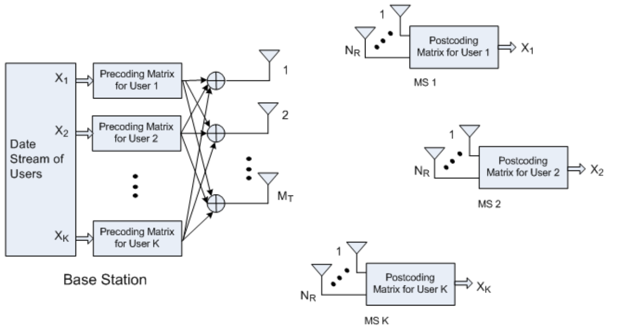

Figure 1. Multiuser MIMO Downlink system model

paper, we propose downlink precoding technique in which not only service requirement of each user is attained but also total sum rates is increased in common Signal to Noise Ratios (SNRs).

This paper is organized as follows. In section II, we describe system model and review BD and SO pre-coding schemes. In section III, the proposed downlink precoding scheme is developed. In section IV, simulation results are presented and finally the paper is concluded in section V.

II. SYSTEM MODEL

In this section, first we describe the MIMO downlink channel and signals model, then we explain Block Diagonalization and Successive Optimization precoding schemes in more detail, since they are the building blocks of our precoding method.

A. MIMO downlink signaling

Consider the downlink channel of MIMO multiuser system with $\mathbf{M}_T$ transmit antennas and K users, each employing $\mathbf{N}_k$ antennas at the receiver as shown in figure 1.

Let $\mathbf{H}_k$ Denotes the channel matrix of user k. In this paper we assume flat fading Rayleigh environment, thus elements of $\mathbf{H}_k, k = 1, \ldots, K$ can be modeled as Independent Identical Distributed (i.i.d.) complex Gaussian random variables with zero man and unite variance. Let $\mathbf{x}_k$ shows the transmit signal vector of user k and $\mathbf{n}_k$ is the vector of additive white complex Gaussian thermal noise with covariance matrix equal to $\sigma^2\mathbf{I}$. The signal of each user preprocessed at transmitter by pre-coding matrix; Therefore, at mobile station the received signal of user k is given as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{M}_k \mathbf{x}_k + \mathbf{H}_k \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{M}_k \mathbf{x}_k + \mathbf{n}_k \tag{1}$$

where $\mathbf{M}_k$ denote the precoding matrix of user k. The second part of (1) on the right side is interference result from messages belong to other users. In order to transmit different messages to different users at the same time using spatial multiplexing, number of base station antennas must be greater or equal to $\sum_{k=1}^{K} \mathbf{N}_k$ [8]. For simplicity, we assume that all mobile stations have the same number of receive antennas $\mathbf{N}_R$, which is equal to number of each user data streams.

B. Block Diagonalization

In BD scheme, precoding matrix $\mathbf{M}_k$ of k^{th} user lies in the null space of aggregate matrix channel of other users, which is defined as

$$\overline{\mathbf{H}}_k = [\mathbf{H}_1^T, \ldots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \ldots, \mathbf{H}_K^T]^T \tag{2}$$

so we should have $\mathbf{H}_l \mathbf{M}_k = \mathbf{0}$ for $l \neq k$. This means interference of other users is totally eliminated for the k^{th} user. Let us denote SVD decomposition of matrix $\overline{\mathbf{H}}_k$ by λ

$$\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_k \overline{\mathbf{\Lambda}}_k [\overline{\mathbf{V}}_k^{(1)}\ \overline{\mathbf{V}}_k^{(0)}]^H \tag{3}$$

where $\overline{\mathbf{U}}_k$ is the left singular matrix of $\overline{\mathbf{H}}_k$, and $\overline{\mathbf{\Lambda}}_k$ is diagonal matrix whose diagonal elements are singular values of $\overline{\mathbf{H}}_k$. Sub-matrices $\overline{\mathbf{V}}_k^{(1)}$ and $\overline{\mathbf{V}}_k^{(0)}$ are the right singular matrices corresponding to nonzero and zero singular values of $\overline{\mathbf{H}}_k$ respectively; therefore, $\mathbf{H}_k \overline{\mathbf{V}}_k^{(0)} = \mathbf{0}$ for all $l \neq k$. Suppose that fading among users and between antennas of each user are independent, so all matrices would be full rank, thus there exists matrix $\overline{\mathbf{V}}_k^{(0)}$ with $\mathbf{N}_k$ columns which is formed the null space basis for $\overline{\mathbf{H}}_k$. Now the precoding matrix can be constructed such that it satisfies the zero interference between users [8].

After removing interference between users the MIMO downlink channel is decomposed into K parallel non-interfering single user channels where the received signal at k^{th} user is given by

$$\mathbf{y}_k = \mathbf{H}_{eff,k} \mathbf{x}_k + \mathbf{n}_k \tag{4}$$

where $\mathbf{H}_{eff,k}$ is effective channel for k^{th} user and it equals to $\mathbf{H}_k \mathbf{M}_k$. Now the throughput maximization turns into individual rate maximization for each user. The solution of this problem is obtained by choosing the precoding matrix as the right singular value of $\mathbf{H}_{eff,k}$ and employing well known water-filling algorithm on its singular values to achieve maximum rate [13]. let denote the SVD decomposition of effective channel by

$$\mathbf{H}_{eff,k} = \mathbf{U}_k [\mathbf{\Lambda}_k\ \mathbf{0}][\mathbf{V}_k^{(1)}\ \mathbf{V}_k^{(0)}]^H \tag{5}$$

where $\mathbf{V}_k^{(1)}$ are the right singular vectors corresponding to non-zero singular values and $\mathbf{U}_k$ are the left singular vectors. Therefore the final precoding matrix of k^{th} user is equal to $\mathbf{M}_k = \overline{\mathbf{V}}_k^{(0)} \mathbf{V}_k^{(1)}$ which means that we first eliminate interference of other users, and then transform the user channel into parallel channels, to optimize power allocation. The decoding matrix would be $\mathbf{U}_k$ at the receiver of k^{th} user. Now the maximum achievable rate is obtained by allocating power, using water-filling algorithm on singular values of all users as follow

$$C_{BD} = \sum_{i=1}^{K} \log_2 |\mathbf{I} + \frac{\mathbf{\Lambda}_k^2 \mathbf{\Sigma}_k}{\sigma_n^2}| \tag{6}$$

Here $|\cdot|$ refers to the determinant operator. In the above equation $\mathbf{\Sigma}_k$ denote diagonal matrix of optimum power loading coefficients, subjected to the total power constraint, which is obtained by using water-filling algorithm over singular values of all users. $\mathbf{\Lambda}_k$ is the diagonal matrix of singular values for user k.

C. Successive Optimization

The aim of SO algorithm is to solve minimization problem of transmit power and achieve the required target rate for each user. In this scheme, one user is precoded at a time, such that it does not interfere with previously encoded users. In order to achieve this goal, precoding matrix of k^{th} user lies in the null space of aggregate channel of users $i = 1, \dots, k-1$, which these users are precoded before k^{th} user. Then the power allocation is performed individually for each user by taking into account the interference from $k-1$ previously encoded users and satisfying the desired rate for target user [12]. The aggregate channel of previously encoded users is given by

$$\mathbf{H}_k = [\mathbf{H}_1^T, \dots, \mathbf{H}_{k-1}^T]^T \tag{7}$$

Like BD approach, we denote the right singular vectors of the null space of $\mathbf{H}_k$ by $\mathbf{V}_k^{(0)}$. Then the precoding matrix would be $\mathbf{M}_k = \mathbf{V}_k^{(0)} \mathbf{M}'_k$; which is the linear combination of $\mathbf{V}_k^{(0)}$ and some choice of $\mathbf{M}'_k$. Also the noise plus interference covariance matrix for k^{th} user is

$$\mathbf{Q}_k = \sigma_n^2 \mathbf{I} + \sum_{i=1}^{k-1} \mathbf{H}_k \mathbf{M}_i \mathbf{M}_i^H \mathbf{H}_k^H \tag{8}$$

The objective of SO is to find $\mathbf{M}'_k$ such that $trace(\mathbf{M}'_k \mathbf{M}'^H_k)$ is minimized [12] (here $trace(\cdot)$ is the sum of the elements on the matrix main diagonal). In this case the achieved required rate for k^{th} user is

$$R_k = \log_2 |\mathbf{I} + \mathbf{M}'^H_k \mathbf{V}_k^{(0)H} \mathbf{H}_k^H (\mathbf{Q}_k^{-1}) \mathbf{H}_k \mathbf{V}_k^{(0)} \mathbf{M}'_k| \tag{9}$$

This leads to a water-filling solution in MIMO link with co-channel interference [13], using the following SVD decomposition

$$\mathbf{V}_k^{(0)H} \mathbf{H}_k^H (\mathbf{Q}_k^{-1}) \mathbf{H}_k \mathbf{V}_k^{(0)} = \mathbf{W}_k \mathbf{\Lambda}'_k \mathbf{W}_k^H \tag{10}$$

The precodeing matrix would be $\mathbf{M}_i = \mathbf{V}_k^{(0)} \mathbf{W}_k \mathbf{\Sigma}'_k$ where $\mathbf{\Sigma}'_k$ is the power allocation diagonal matrix with elements achieved by performing water-filling algorithm on the elements of $\mathbf{\Lambda}'$. The total transmit power is the sum of the elements of all $\mathbf{\Sigma}'_k$ for all users. The power minimization problem depends on the order in which users are precoded [12].

III. SUCCESSIVE BLOCK DIAGONALIZATION (SBD) PRE-CODING

The BD method treats all users the same, from spatial domain resource allocation point of view; therefore, with equal transmit power and power decay due to the channel path loss for each user, ergodic transmit rate for all users will be equal. In Block Diagonalization method with fixed total transmit power constraint, we can change the proportional rate between users by allocating different transmit power to them, but in cost of considerable lost in total sum rate.

On the other hand, SO can assign different rats among users according to required rate for each one. However, with the same transmit power and path loss for each user, the user that is precoded later in OS achieves lower bit rate compare to users which were precoded earlier in the process [12]. If the number of users increase, the users that processed at the end will get a small portion of channel capacity. In order to prioritize the different service levels in system, and achieve the large part of channel sum capacity as the same time, we propose Successive Block Diagonalization (SBD) algorithm as follow:

First we divide users to different groups based on their rate requirements by putting users with the same rate requirement, in different groups. It means that users that are in the same group will get different rates based on their service levels. let denote aggregate channel for all users in m^{th} group as

$$\mathbf{H}_{G_m} = [\mathbf{H}_1^T, \dots, \mathbf{H}_{S_m}^T]^T \tag{11}$$

In above equation S_m is the number of users in m^{th} group, and the total number of users is $S = \sum_{m=1}^{L} S_m$ where L is the total number of groups. $\mathbf{H}_k$ is channel of user k in the group. In first step we will perform the Block Diagonalization on all of these aggregate channels, each of them can be seen as an effective channel of the group. If we perform BD on these effective group channels, the interference between groups will be removed. We show the aggregate channel of all groups beside the m^{th} group by

$$\overline{\mathbf{H}}_{G_m} = [\mathbf{H}_{G_1}^T, \dots, \mathbf{H}_{G_{m-1}}^T, \mathbf{H}_{G_{m+1}}^T, \dots, \mathbf{H}_{G_L}^T]^T \tag{12}$$

As we mentioned before in BD algorithm, the null space of $\overline{\mathbf{H}}_{G_m}$ is obtained by SVD decomposition and take the right singular vectors of zero singular values of this matrix. If $\overline{\mathbf{V}}_{G_m}^{(0)}$ indicate the null space vectors of $\overline{\mathbf{H}}_{G_m}$, then the effective channel of m^{th} group is

$$\mathbf{H}_{G_m}^{\mathit{eff}} = \mathbf{H}_{G_m} \overline{\mathbf{V}}_{G_m}^{(0)} \tag{13}$$

The effective channel columns of each user in $\mathbf{H}_{G_m}^{\mathit{eff}}$ are corresponding to the same columns in $\mathbf{H}_{G_m}$. We show the effective channel matrix of m^{th} group as follow

$$\mathbf{H}_{G_m}^{\mathit{eff}} = [\mathbf{H}'^T_{1,G_m}, \dots, \mathbf{H}'^T_{S_m,G_m}]^T \tag{14}$$

where $\mathbf{H}'^T_{k,G_m}$ is the effective channel of k^{th} user in group m. In the second step we perform the successive optimization algorithm on the users effective channel in each group. The order of successive optimization is based on the rate requirements in which the user with higher rate level precoded before the user with lower rate level. The rate requirement for each user can be calculated through equations (7)-(9), by using the effective channel of that user which is achieved in (14). Now assume that aggregate channel of previously processed users before k^{th} user in group m is

$$\mathbf{H}'_{k,G_m} = [\mathbf{H}'^{T}_{1,G_m}, \ldots, \mathbf{H}'^{T}_{k-1,G_m}]^T \tag{15}$$

And we show the null space of $\mathbf{H}'_{k,G_m}$ by $\mathbf{V}^{(0)}_{k,G_m}$. Now the precoding matrix for this user would be $\mathbf{M}_{k,G_m} = \mathbf{V}^{(0)}_{k,G_m}\mathbf{M}'_{k,G_m}$. While the interference of users in other groups totally cancelled by BD at last step, we can write the noise plus interference for k^{th} user in group m as

$$\mathbf{Q}_{k,G_m} = \sigma_n^2\mathbf{I} + \sum_{i=1}^{k-1}\mathbf{H}'_{i,G_m}\mathbf{M}_{i,G_m}\mathbf{M}^H_{i,G_m}\mathbf{H}'^H_{i,G_m} \tag{16}$$

and $\mathbf{M}'_{k,G_m}$ is calculated as we discussed for SO algorithm. Then the rate of user k, in group m would be equal to

$$\begin{aligned} R_{k,G_m} = \log_2 |\mathbf{I} + \mathbf{M}'^H_{k,G_m}\mathbf{V}^{(0)H}_{k,G_m} \\ \mathbf{H}'^H_{k,G_m}(\mathbf{Q}^{-1}_{k,G_m})\mathbf{H}'_{k,G_m}\mathbf{V}^{(0)}_{k,G_m}\mathbf{M}'_{k,G_m}| \end{aligned} \tag{17}$$

In this algorithm, the user which is precoded first in each group get largest amount of capacity in the group (assuming equal power allocation among users), The second precoded user achieves less capacity than first user but larger than the others and so on and so forth. Therefore users that require higher service level will be put in different groups and precoded first in SO algorithm. Users that require the second service level are put in different groups and precoded in second step, and so on. Therefore the users with the same service level distributed in different groups, which are called service groups.

The required rate for users can be calculated by (17); hence we can determine the amount of power for each service in advanced, and share the power between service groups based on the required rate for that service.

IV. Simulation Results

In this section we compare the sum achievable rates of proposed scheme i.e. successive block diagonalization with traditional BD and SO precodings, through Monte Carlo Simulation which is done for 10000 channel realization. We assume that the fading channel elements of each user are independent complex Gaussian random variables with zero mean and unit variance.

In the simulations, we assumed multiuser downlink MIMO with 8 antennas at base station and 4 users each of them employ two antennas. Our aim is to prepare two different service levels for two groups of users, each of them contains two users. Also we assume that users 1 and 2 require service level with higher data rate and users 3 and 4 require service with lower data rate.

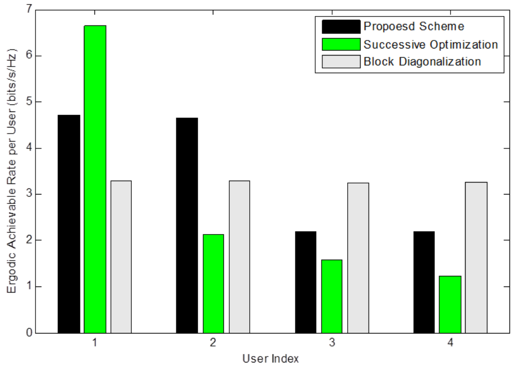

Figure 2. The Comparison of Ergodic Achievable Sum Rate per user for Proposed scheme, BD and SO methods when total power is shared equally between two service groups (SNR=10dB)

Figure 2 shows ergodic sum rate distribution among users provided that total power is equally shared between two groups. From this figure we observe that in this case, the BD method shares the sum rate equally between users and it is not possible to set various services for different users.

The SO algorithm can assign different rate levels among users, but users that is precoded later obtain a small portion of total capacity. Also in SO, with equal power sharing between users, large part of capacity is allocated to the first user, which is not reasonable in practical systems. We can see in the figure that proposed algorithm not only can set the different rates between groups to achieve required rates; but also guarantee the fairness among users in each service group, which is required to get the same rate for these users. In order to achieve the desired rates for each user, first equations (8) and (16) should be solved, then the required powers should be determined. On the other hand, to assign the different rates between each service, we can distribute total power with different proportion among service groups. Figure 3 shows the Ergodic sum rate distribution among users. Here the power is allocated between two service groups with ratio equal to 3:1. As we can observe from figure, BD cannot set proper rate ratios between two groups but proposed scheme can offer a wide range of rate ratio among service groups while preserving the maximum achievable sum rate.

Figure 4 illustrates the total Ergodic sum capacity of different schemes versus Signal to Noise Ratio (SNR). We observe that for considerable portion of SNR level, the ergodic sum rate of proposed scheme, i.e. Successive Block Diagonalization (SBD), achieves a better performance than Block Diagonalization ,except for high SNRs, and Successive Optimization ,except for low SNRs.

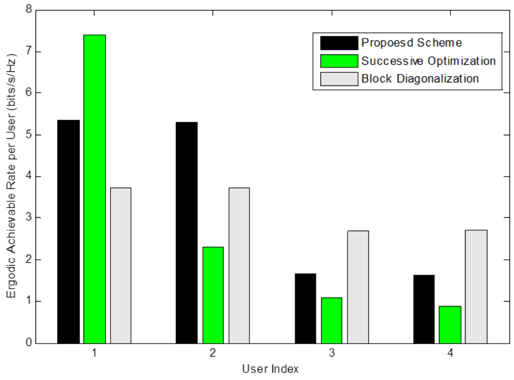

Figure 3. The Comparison of Ergodic Achievable Sum Rate per user for Proposed scheme, BD and SO methods; total power is shared with the ratio of 3:1 between two service groups (SNR=10dB)

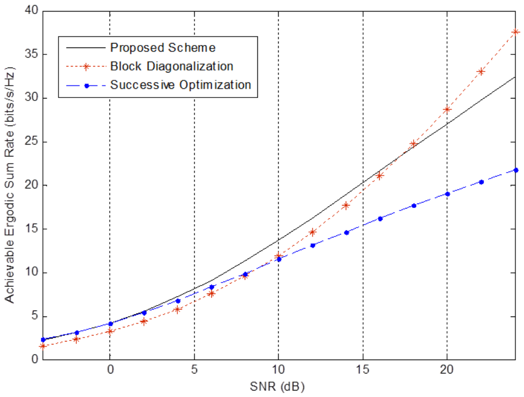

Figure 4. Ergodic Sum Rate for Proposed scheme, BD and SO methods when total power is shared with the ratio of 3:1 between two service groups

Also we can see that the total rate difference between SBD and two conventional methods in the low and high SNRs is not comparable. Therefore we can get different required rates for different services with better sum rate in common SNRs and without comparable loss in other SNRs.

V. CONCLUSION

In this paper, we proposed a downlink MIMO transmission scheme based on spatial linear precoding methods to prepare different service levels for users while preserving the achievable sum rate of the channel. The proposed algorithm uses Successive Optimization scheme to determine the different rates for different service levels and prioritize the users, and exploit the Block Diagonalizaiton method to remove the interference between groups. The simulation results showed that the proposed algorithm can set different proportional rate between service groups and achieve desired rates with proper power loading. Simulations also verified that proposed algorithm outperform the BD and SO sum rates in the common SNRs and achieve large portion of their achievable rate in other SNR levels.

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