



Finite Word Length Effects on Two Integer Discrete Wavelet Transform Algorithms

Armein Z. R. Langi

ITB Research Center on Information and Communication Technology
DSP-RTG, Information Technology Research Division
School of Electrical Engineering and Informatics
ITB Microelectronic Center
Institut Teknologi Bandung, Jalan Ganeca 10 Bandung, Indonesia 40132

Abstract: This paper studies finite word-length effects on two different VLSI architectures of integer discrete wavelet transforms (DWT). The two DWT architectures representing two extreme cases are Scheme 1: basis correlation, and Scheme 2: pyramidal algorithm. For signal-to-noise ratio (SNR) evaluations, we consider various values of the length of integer word (W). Our experiments show that W is critical for both schemes, although both schemes perform almost equivalently. We also show that Scheme 1 and Scheme 2 have computational complexities $O(N^2)$ and $O(N)$, respectively. The paper concludes that the word length W has similar critical impacts on the quality of integer DWT of both Schemes, hence Scheme 2 should be used based on lower computational complexity reason.

Keywords: VLSI Architecture, word-length effects, DWT, Pyramidal Algorithm.

1. Introduction

This paper studies finite word-length effects on two different VLSI architectures for integer discrete wavelet transforms (DWT). The DWT has become increasingly important in fields such as digital signal processing, speech and audio processing, and image and video processing [1], to provide multi-scale temporal-spectral analysis. Consequently, VLSI implementations are often required.

This work is a part of our design of a DWT processor for a speech compression scheme described in [2]. The design uses VHDL and high-level synthesis as design tools, with field-programmable gate array (FPGA) as the target technology [3]. The designed is constrained to the more-efficient integer multipliers. Using iterative array of cells for partial products reduction, a 16×16 -bit multiplier requires four 8×8 -bit multipliers such as 74S557 [4].

In our design, we consider of using the usual pyramidal algorithm. However since it is recursive in nature, we concern the impact of using finite integer on its performance. In specific, computational errors introduced by finite data word lengths may propagate recursively. As a result, the quality of DWT may deteriorate rapidly, causing the results to be unusable. We should then study the impact of finite word lengths on pyramidal algorithm and compare the results with those of a non-recursive DWT algorithm.

We consider two DWT algorithms representing two extreme cases: (i) basis correlation, and (ii) pyramidal algorithm [5], [6]. The basis correlation scheme (Scheme 1) produces DWT results from inner-products of the input vector with a set of wavelet basis signals, while the pyramidal scheme (Scheme 2) obtains the results by recursively filtering the input signal using wavelet and scaling filters. The basis signals and the filter's impulse responses come from a wavelet prototype.

This paper, expanding a description of our work reported in [7], is organized as follow. First, Section II describes DWT algorithms, defines both Scheme 1 and Scheme 2, and

showing that Scheme 2 is recursive in nature. Section III presents the experiments of integer word-length impacts on algorithms' quality, showing that both schemes behave similarly under various computational conditions. Section IV discusses results of complexity analysis of both schemes, shows the benefits of Scheme 2, and finally concludes that Scheme 2 should be used as the basic algorithm for DWT VLSI architectures.

2. The DWT Algorithms

In a usual vector setting, a signal can be represented by a vector \mathbf{x} in a Euclidean vector space. If the vector space is M -dimensional, having orthonormal basis vectors $\{\mathbf{u}_1, \dots, \mathbf{u}_M\}$, the signal can be represented by

$$\mathbf{x} = \sum_{i=1}^M \alpha_i \mathbf{u}_i \quad (1)$$

Where the coefficients α_i are obtained using an inner product

$$\alpha_i = \langle \mathbf{x}, \mathbf{u}_i \rangle \quad (2)$$

In a case of digital signals $x[n]$ defined in a l^2 vector space, we have orthonormal basis signals $\{u_1[n], \dots, u_M[n]\}$ such that

$$x[n] = \sum_{i=1}^M \alpha_i u_i[n] \quad (3)$$

If $u_i^*[n]$ is the complex-conjugate of $u_i[n]$, the inner product is obtained using a form of basis correlation

$$\alpha_i = \sum_{n=1}^N x[n] u_i^*[n] \quad (4)$$

Now we can apply the concept to understand the basis correlation algorithm of DWT.

A. The Basis Correlation DWT Algorithm

For a given mother wavelet $\{\phi[n], \psi[n]\}$, one can define a set of scaling and wavelet functions recursively [8], namely

$$\begin{aligned} \phi_{j,k}[n] &= \sqrt{2^{-j}} \phi[2^{-j}n - k] \\ \psi_{j,k}[n] &= \sqrt{2^{-j}} \psi[2^{-j}n - k] \end{aligned} \quad (5)$$

Let us define $j = 1, \dots, J$, where $J = (\log_2 N) - 1$, and k depends on j , i.e., $k(j) = 0, \dots, 2^{-j}N - 1$. There will be $M - 1$ wavelet functions $\{\psi_{j,k(j)}[n]\}$ and one scaling function $\{\phi_{j,k(j)}[n]\}$. We can then combine $\{\psi_{j,k(j)}[n]\}$ and $\{\phi_{j,k(j)}[n]\}$ as a set of orthonormal basis signals, in which there is a one-to-one mapping $(j, k(j)) \leftrightarrow i$, such that the set of all $\psi_{j,k(j)}[n]$ corresponds to $\{u_1[n], \dots, u_{M-1}[n]\}$ and $u_M[n] = \phi_{j,k(j)}[n]$.

For a given set of input samples $x[n]$, $n = 0, \dots, N - 1$, we can define DWT to be a set of coefficients $\{c_{j,k(j)}, d_{j,k(j)}\}$ according to Eq. (4):

$$\begin{aligned} c_{j,k(j)} &= \sum_{n=0}^{N-1} x[n] \psi_{j,k(j)}^*[n] \\ d_{j,k(j)} &= \sum_{n=0}^{N-1} x[n] \phi_{j,k(j)}^*[n] \end{aligned} \quad (6)$$

such that

$$x[n] = \sum_{j=1}^J \sum_{k(j)=0}^{2^{-j}N-1} c_{j,k(j)} \psi_{j,k(j)}[n] + \sum_{k=0}^{2^{-J}N-1} d_{J,k(J)} \phi_{J,k(J)}[n] \quad (7)$$

Scheme 1 computes wavelet coefficients $\{c_{j,k(j)}, d_{j,k(j)}\}$ directly using Eq. (6). Here we constructs N basis signals consisting of $\{\psi_{j,k(j)}[n]\}$ and $\{\phi_{j,k(j)}[n]\}$. Implementing Eq. 4, Eq. (6) defines inner-product of signal input samples $x[n]$ with those basis signals. Consequently, Eq. (7) implements Eq. (1).

B. Pyramidal DWT Algorithm

It is well known that for orthonormal wavelet, both signals $\{\phi[n], \psi[n]\}$ are closely related. Consider for example Daubechies wavelets [9]. Table I shows scaling coefficients $\phi[n]$ for

Daubechies wavelets of lengths $L = 4, 12,$ and 20 . Corresponding wavelet coefficients are derived from the scaling coefficients according to

$$\psi[n] = (-1)^{-n} \phi[L - 1 - n] \quad (8)$$

This relation in Eq. (8) ensures the basis signals are orthonormal, and both $\{\psi_{j,k(j)}[n]\}$ and $\{\phi_{j,k(j)}[n]\}$ can be derived from $\phi[n]$ in Table I using Eq. (5). Figure 1 shows mother wavelets $\psi[n]$ for $L = 4, 12,$ and 20 .

Table 1
Scaling Coefficients of Daubechies Wavelets [9].

n	Scaling Coefficients $\phi[n]$ for Various L		
	$L = 4$	$L = 12$	$L = 20$
0	0.6830127	0.15774243	0.03771716
1	1.1830127	0.69950381	0.26612218
2	0.3169873	1.06226376	0.74557507
3	-0.1830127	0.44583132	0.97362811
4		-0.31998660	0.39763774
5		-0.18351806	-0.35333620
6		0.13788809	-0.27710988
7		0.03892321	0.18012745
8		-0.04466375	0.13160299
9		7.83251152e-4	-0.10096657
10		6.75606236e-3	-0.04165925
11		-1.52353381e-3	0.04696981
12			5.10043697e-3
13			-0.01517900
14			1.97332536e-3
15			2.81768659e-3
16			-9.69947840e-4
17			-1.64709006e-4
18			1.32354367e-4
19			-1.875841e-5

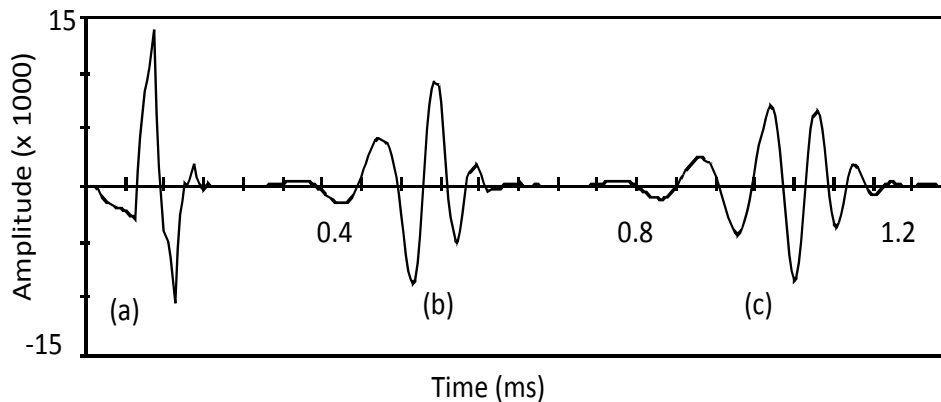


Figure 1. Daubechies mother wavelets $\psi[n]$, for (a) $L = 4$, (b) $L = 12$, and (c) $L = 20$.

As a result of such a close relationship in Eq. (8), Eq (6) can be implemented in a pyramidal structure of filterbanks, having impulse responses corresponding to wavelet and scaling functions. There are two filters: highpass filter (HPF) and lowpass filter (LPF). Both the HPF

and LPF are of finite impulse response (FIR), having impulse responses corresponding to scaling coefficients and wavelet coefficients, respectively.

Scheme 2 has the following algorithm shown a pseudocode in Figure1. At the first phase, $j = 1$, those N input samples $x[n]$ are passed through a HPF and an LPF, simultaneously, resulting in N samples $y_h[n]$ and $y_l[n]$, respectively (see also Fig 3). The scheme subsamples-by-2 the $y_h[n]$ to be $c_{1,k}$, i.e., $c_{1,k} = y_h[2k + 1]$. Furthermore, $d_{1,k} = y_l[2k + 1]$. Thus there are $N/2$ samples of $c_{1,k}$ and $d_{1,k}$.

- (1) INITIALIZE $d_{0,k}[n] = x[n]$, FOR $n = 0, \dots, N - 1$
- (2) SET $J = (\log_2 N) - 1$; $K = N$;
- (3) FOR $j=1$ TO J , DO LOOP 1
- (4) $K = K / 2$;
- (5) FOR $k = 0$ TO $K - 1$, DO LOOP 2
- (6)
$$c_{j,k} = \sum_{i=0}^{L-1} (-1)^i \phi_{L-1-i} d_{j-1,2k+i}$$
- (7)
$$d_{j,k} = \sum_{i=0}^{L-1} (-1)^i \phi_i d_{j-1,2k+i}$$
- (8) END LOOP 2
- (9) END LOOP 1

Figure 2. A pseudocode of the DWT pyramidal algorithm for L point wavelet.

Now, for the second phase, $j = 2$, the scheme repeats the process. It takes $N/2$ samples $d_{1,k}$ to be used as input of the HPF and LPF, simultaneously, resulting in $N/2$ samples $y_h[n]$ and $y_l[n]$, respectively. The scheme then subsamples-by-2 the the $y_h[n]$ to be $c_{2,k}$, i.e., $c_{2,k} = y_h[2k + 1]$. Furthermore $d_{2,k} = y_l[2k + 1]$. Thus there are $N/4$ values of $c_{2,k}$ and $d_{2,k}$. The process is repeated for the next j until $j = J$, where at each stage j , the input is $N/(2^{j+1})$ samples $d_{j-1,k}$ to both HPF and LPF, simultaneously, to produce $N/(2^{j+1})$ samples $y_h[n]$ and $y_l[n]$ (see Figure3). It then subsamples-by-2 the $y_h[n]$ to be $c_{j,k}$, i.e., $c_{j,k} = y_h[2k + 1]$. Furthermore $d_{j,k} = y_l[2k + 1]$. At the end of the algorithm, after $j = J$, we have all wavelet coefficients $\{c_{j,k}, d_{j,k}\}$ as desired.

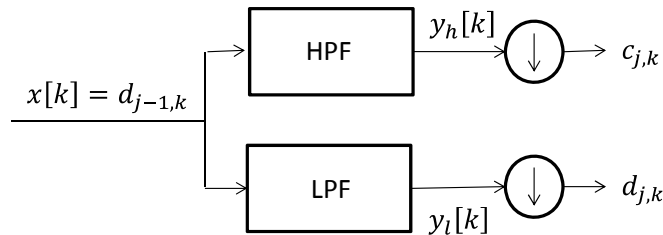


Figure 3. Filtering for the pyramidal algorithm at phase j .

To illustrate the use of filtering in Figure3 for DWT, consider a sample signal $x[n]$, shown in Figure4. Here we use $N = 64$. As a result, $J = 5$, and we have five recursive filtering phases $j = 1, \dots, 5$. The first phase results in two signals: wavelet highpass signals at $j = 1$ (see Fig 5.a), and scaling low pass signal at $j = 1$. This scaling low pass signal is used as the input for the next phase $j = 2$, resulting in wavelet high pass signals at $j = 2$ (see Fig 5.b), and scaling low pass signal at $j = 2$. This scaling low pass signal becomes the next phase input ($j = 3$),

resulting in wavelet high pass signals at $j = 3$ (see Fig 5.c), and scaling low pass signal at $j = 3$. Similar filtering at $j = 4$, resulting in resulting in wavelet high pass signals at $j = 4$ (see Fig 5.d), and scaling low pass signal at $j = 4$. Finally, filtering of the scaling lowpass signal at $j = 5$, resulting in wavelet high pass signals at $j = 5$ (see Fig 5.e), and scaling low pass signal at $j = 5$ (see Fig 5.f). It should be noted that if we sum all these filtering signals (Fig 5.a to f), we will have exactly $x[n]$ in Figure 4.

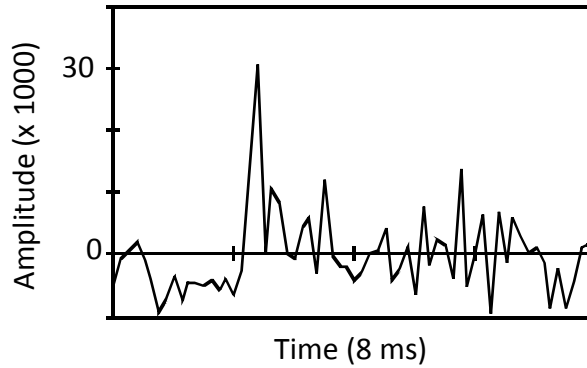
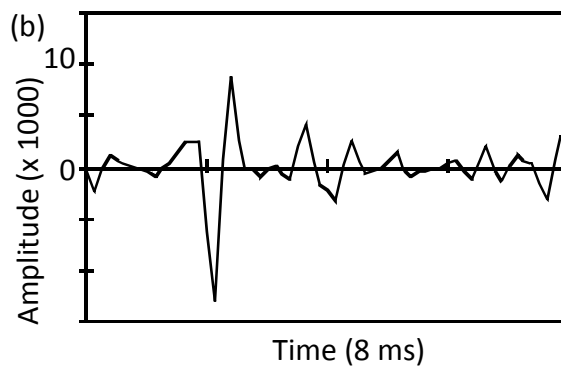
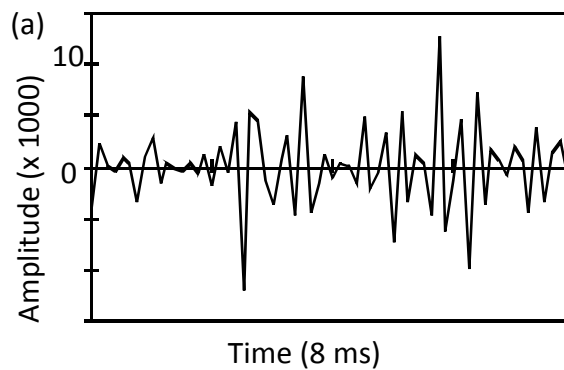


Figure 4. An 8-ms block of sample signal $x[n]$.



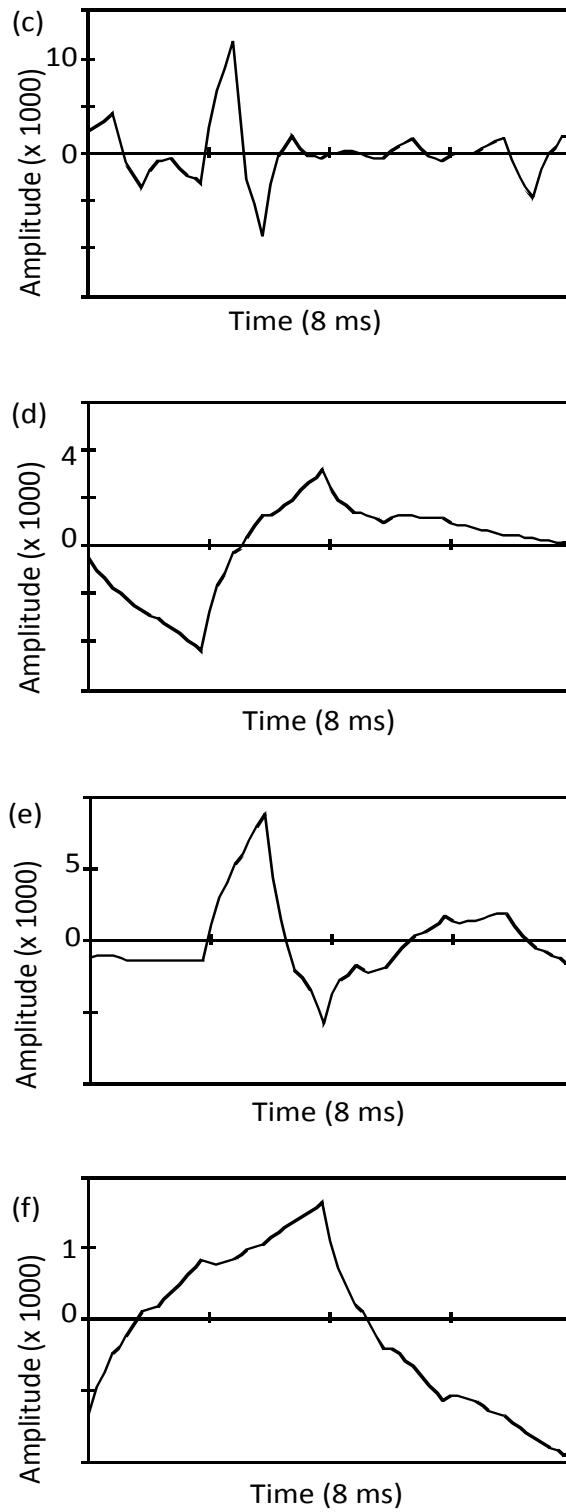


Figure 5. An 8-ms block of wavelet filtering results of sample signal $x[n]$ in Figure1, where (a) wavelet high pass at $j = 1$, (b) wavelet high pass at $j = 2$, (c) wavelet high pass at $j = 3$, (d) wavelet high pass at $j = 4$, (e) wavelet high pass at $j = 5$, and (f) scaling low pass at $j = 5$.

3. Word Length Effects

We expect the computational structure of pyramidal algorithm is more efficient comparing to that of basis correlation scheme. As a result, Scheme 2 should be the choice of VLSI architecture. However an efficient VLSI architecture requires integer implementations. In general the quality of integer architecture is sensitive to word length. Our concern with Scheme 2 is it involves a pyramidal structure, hence it is recursive. In recursive cases, the arithmetic word-length becomes an important issue.

We then study the performance of both Scheme 1 and Scheme 2 under integer arithmetics. The performance of the schemes is evaluated according to the length of integer word (W), at specified length of input samples (N) and the length of wavelet prototype (L). Notice that N represents the number of input samples, taking integer values of power of 2. We limit L to 4, 12, and 20, to cover Daubechies prototype wavelets of length 4, 12, and 20 [9] (see also Figure1). Finally, W should cover the usual integer word lengths of 8, 14, 16, 24, and 32. For completeness, W is varied from 4 to 32 .

Here as shown in Figure6, we first apply uniformly distributed random samples as input signal $x[n]$ to both schemes, resulting in respective coefficients $\{c_{j,k}[n], d_{j,k}[n]\}$. We then apply both results independently to an inverse DWT to produce reconstructed signals $\hat{x}[n]$. Finally we compare the resulting signals $\hat{x}[n]$ with the original ones $x[n]$, and measure the signal-to-noise ratio (SNR) of the re-synthesized signals, according to

$$SNR = 10 \log_{10} \frac{\sum_{n=0}^{N-1} |x[n]|^2}{\sum_{n=0}^{N-1} |\hat{x}[n] - x[n]|^2} \quad (4)$$

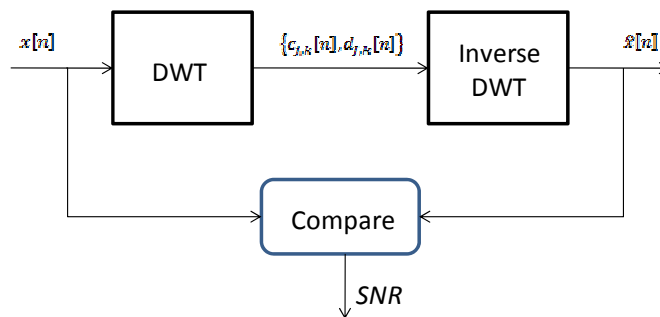


Figure 6. Measuring quality of the Schemes.

Table II shows the SNR as a function of W . Our experiments to assess round-off effects show that W is critical for both schemes. Changing W will change the SNR dramatically (see Figure7). In some signal applications, an SNR level of 30dB is considered minimal. Thus an integer DWT must use at least $W = 12$ bits. For word length of 16 bit, the SNR is already at an excellent level of 61 dB. And the integer DWT at 32 bit performs overwhelmingly well. However notice that both schemes perform almost equivalently, and in most cases Scheme 2 seems to outperform Scheme 1, illustrated in Figure7. It seems that the recursive nature of the pyramidal algorithm does not propagate round-off errors. The two schemes behave similarly due to round-off effects. The most important point is that this means there is no advantage in SNR of using Scheme 1. It should be noted that in our experiment N and L have no significant effects on the SNR.

Table I
Effect of word length W to SNR, for $N = 1024$ and $L = 20$.

W	Signal to Noise Ratio (dB)	
	Scheme 1	Scheme 2
4	1.60	1.48
8	16.27	16.76
12	37.77	38.98
14	49.23	50.93
16	61.36	62.71
24	108.94	111.27
32	157.24	159.58

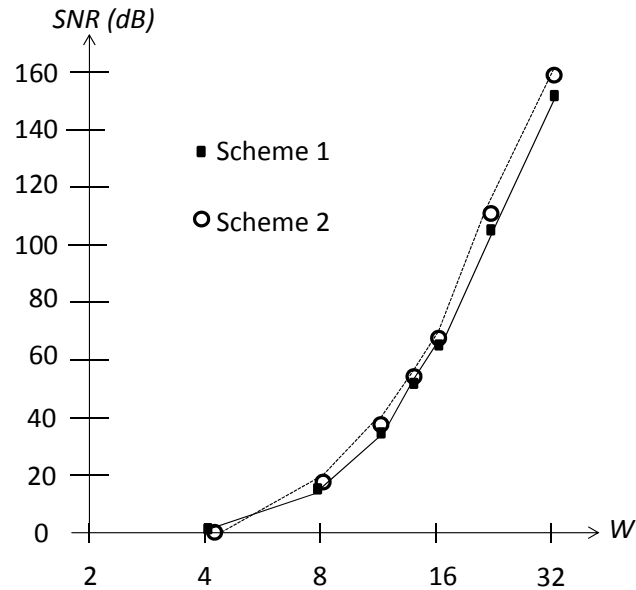


Figure 7. Critical impacts of word length W to the quality.

4. Discussions and Concluding Remarks

We have compared two different algorithm candidates of VLSI architectures for integer DWT, namely basis correlation (Scheme 1) and pyramidal algorithm (Scheme 2). Scheme 1 has a direct relationship with the DWT definition, hence it is expected to perform well in SNR using integer computations. On the other hand Scheme 2 is of recursive nature, resulting in potential accumulating computational error propagations.

However, our experiments show that Scheme 2 is as computationally good as Scheme 1, i.e., SNRs for various word lengths are comparable. Furthermore, the behavior of both scheme are comparable for various sample lengths or wavelet prototype lengths. This means Scheme 1 has no advantages over Scheme 2.

Scheme 1 has a computational structure directly following Eq (2), hence it has a simpler and straightforward control structure. However by observing the equation, we conclude that for each coefficient in Eq. (6) there are N multiplications and $N-1$ accumulations to be made. Since there are N coefficients to be produced, we can say that the complexity of Scheme 1 is $O(N^2)$. In fact, the total computation is found to be $N^2 + N \log_2(N/L) + N - L$.

As expected, Scheme 2 requires fewer computational operations (i.e., multiplication and addition). In particular each coefficient requires L multiplications and $L-1$ accumulations because of LPF and HPF. Furthermore, there are N coefficients to be produced by HPF and N

coefficients by LPF. Hence, Scheme 2 has $2NL$ total computation, or simply $O(N)$ as opposed to $O(N)^2$ in Scheme 1.

Thus for a typical N and L of 64 and 4, respectively, Schemes 1 and 2 result in 3900 and 512 operations, respectively. It should be noted that Scheme 1 uses very simple processing elements (multiply and accumulate) and can lead to very fast, parallel schemes, while Scheme 2 has inherent speed limitation due to its recursive nature. Thus in terms of computational complexity, Scheme 2 outperforms Scheme 1 as expected. Its complexity is only 13.1% of that of Scheme 1 for a typical sample length and a prototype length.

In conclusion, finite word length of data has similar critical impact on DWT performance on both algorithms. We can also conclude that Scheme 2 should be selected as the VLSI architecture of choice in our DWT VLSI design based on computing complexity reason.

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Armein Z. R. Langi, lecturer at STEI-ITB. Ph.D. in Electrical and Computer Engineering, University of Manitoba, Canada, graduated 1996. M.Sc. in Electrical and Computer Engineering, University of Manitoba, Canada, graduated 1992. Ir (B.Sc.) in Electrical Engineering, Bandung Institute of Technology (ITB), Indonesia, graduated 1987. He has papers published in international and national publication.